THE SUMMARY OF Ph. D. DISSERTATION

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Title

Stefan problems in an incompressible viscous fluid flow

Abstract

A free boundary problem which is formulated from the view point of heat conduction to describe the liquid/solid phase transtion phenomena is called the Stefan problem in mathematical physics. If the fluid flow in a liquid medium is neglected, this problem becomes a free boundary problem for the heat equation and has been investigated mathematically since the pioneering study by Stefan (1889). On one hand, problems taken into account the flowing in a liquid medium have been investigated in the framework of weak solutions since 1980, and for the classical solvability, only one result of the existence of the temporally local solution by Bazaliĭ and Degtyarev (1987) is known. However, their formulation is lack of considering

1. the viscous dissipation,
2. the jump of density at the phase change.

Hence we consider more general problem taken into account these effects. That is, the liquid region is assumed to be described by the Navier-Stokes equations and the heat equation with the transport and the viscous dissipation terms, the solid region by the heat equation, and on the liquid/solid interface we impose the conditions derived from the continuities of mass, momentum and energy across the interface. But here, the normal component of the momentum, we replace by the condition that the temperature at the interface is equal to a melting temperature or by the Gibbs-Thompson's law describing the undercooling.

Such a problem has not been discussed in mathematics yet. In this thesis, we prove the existence and uniqueness of the classical solution of the problem.

In chapter 1, we formulate the above problem and introduce necessary function spaces.

In chapter 2, we study 1-phase problem for the liquid phase and prove the existence of a temporally local classical solution. The proof consists of the estimate of the solution of the linearized problem and the solvability of the nonlinear problem. The solution of the linearized problem in a general domain is obtained by summing up the solutions of the linearized Cauchy problems and the initial-boundary value problems in the half space. To the initial-boundary value problem in the half space by making use of the Laplace transform with respect to time and the Fourier transform with respect to the tangential vectors an explicit representation is obtained. The solution in the half space is found as its inverse Fourier-Laplace transformation. This and a fundamental solution yields the representation of the solution to the linearized problem in a general domain. Based on the estimates of Green's function the nonlinear problem is solved by the Schauder's fixed point theorem.

In chapter 3, we study 2-phase problem for both the liquid and the solid phases denote by (P) and prove the existence and the uniqueness of the temporally local classical solution. In the same way as chapter 2, we firstly derive a representation of the solution to the linearized problem by using the Fourier-Laplace transform. But for the 2-phase problem we rely on a result due to Golovkin and Solonnikov on estimates for convolution operators in norms of fractional order. This result makes it possible to estimate Fourier-Laplace transformed functions in Hölder norms. On the basis of this estimate we prove the solvability of the nonlinear problem. By estimating the nonlinear terms more precicely than in chapter 2 a sufficient condition for the contractiveness of them is obtained. From the contractive mapping theorem the uniqness of the solution is concluded. Thus the solution is constructed in the Hölder space with the same Hölder exponent as the data. This is the first result in the multi-dimensional Stefan problems not only for the Navier-Stokes equations but also for the heat equation.

In chapter 4, we study 2-phase problem with the Gibbs-Thompson's law denoted by (P_σ) and prove the existence and the uniqueness of the temporally local classical solution. For the linearized problem we estimate the solution with the same technique used in chapter 3. Since the regularity of the free boundary increases because of the effect of the mean curvature, the contractiveness of the nonlinear terms holds under a weaker condition than that in chapter 3. Under this condition the existence and uniqueness of the solution of the nonlinear problem is proved by the contractive mapping theorem.

In chapter 5, we investigate a relation between the problems in chapters 3 and 4. That is, the solution of problem (P_σ) converges to the solution of problem (P) as the surface tension coefficient σ tends to zero. This is proved by a uniform estimate of the solution of problem (P_σ) with respect to σ. On the basis of this estimate firstly we show that a finite time interval independent of σ can be taken on which problem (P_σ) is uniquely solvable. Secondary, we show that the sequence of solutions is a Cauchy sequence as σ tends to zero.