THE SUMMARY OF Ph. D. DISSERTATION

Major		SURNAME, Firstname
Mathematics		TSUTSUMI, Yukihiro

Title

Algorithmic Methods for Three-Dimensional Topology

Abstract

A knot is a simple closed curve K embedded in a 3-manifold M. From the exterior $E(K) = M - \mathrm{int}N(K)$ and the solid torus $D^2 \times S^1$, by identifying their boundaries with a homeomorphism $f : \partial(D^2 \times S^1) \to \partial N(K)$ we obtain another 3-manifold $\chi(M; K)$ and the operation $M \to \chi(K; M)$ is called Dehn surgery. A Seifert surface for K is an embedded surface $S \subset M$ which is compact, connected, and orientable with $S \cap K = \partial S = K$.

It is well-known that any orientable closed 3-manifold is obtained from S^3 by a finite sequence of Dehn surgeries. In this thesis, we study typical properties of 3-manifolds which are obtained from S^3 by a single Dehn surgery and give several properties of knots in S^3 which distinguish them from those in general 3-manifolds. Here we divide the summary of our main results into geometric part and algebraic part.

Basic tools in studying 3-manifolds from a geometric view point are essential submanifolds. We say a properly embedded surface that is not ∂-parallel is essential if the homomorphism between fundamental groups induced form the inclusion is injective. Haken number of 3-manifold M is the number $h(M)$ which is the upper bound on the number of mutually disjoint, non-parallel essential surfaces in

M. It is known as Haken's finiteness result that $h(M)$ is finite for compact 3-manifolds M. Thus the number of mutually disjoint, non-parallel incompressible Seifert surfaces for a fixed knot has an upper bound. We have obtained the result that a genus one hyperbolic in S^3 bounds at most SEVEN mutually disjoint, non-parallel, genus one Seifert surfaces. At this writing we know an example of hyperbolic knots in S^3 with four such Seifert surfaces. Such a phenomenon can be considered as a difficulty in producing a toroidal 3-manifold with a large number of essential tori from a hyperbolic knot in S^3 and one of typical properties of hyperbolic knots in S^3, even one can generalize this result to general 3-manifolds via Haken numbers.

The Conway polynomial $\nabla_K(z)$ is one of algebraic invariants of knots which is derived from Seifert surfaces. Some restriction to Conway polynomials $\nabla_{K_1}(z)$ and $\nabla_{K_2}(z)$ are known as Casson's formula, where K_1 and K_2 are knots in a homology sphere H_1 such that they yield the same homology sphere H_2. In this thesis, we have studied the case that two knots with distinct Conway polynomials yield the same homology sphere, by giving concrete constructions of such knots.