THE SUMMARY OF Ph. D. DISSERTATION

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Title

The metrical theory of non-archimedean diophantine approximations

(Abstract)

In this thesis, we study the metrical theory of the non-archimedean diophantine approximation. In particular, we consider this problem for formal Laurent power series. This thesis consists of 5 chapters. In Chapters 2 and 3, we discuss about conditions for the diophantine inequality to have infinitely many solutions. In Chapters 4 and 5, we discuss about the convergent rate of some multi-dimensional continued fraction expansions, which give simultaneous approximation sequences. In Chapter 1, we explain some results by Khintchine, Duffin and Schaeffer for the classical case. Also, we give some necessary definitions and show the main results, which are given in the subsequent chapters. In Chapter 2, we consider the question whether $|f - \frac{P}{Q}| < \frac{\psi(Q)}{|Q|}$ has infinitely many solutions or not for a formal Laurent power series f. We discuss the problem when ψ depends on only the degree of Q and when ψ depends on Q itself. When ψ depends on only the degree of Q, we show that $\sum q^n \psi(X^n) = \infty$ is the necessary and sufficient condition for having infinitely many solutions for almost all f. When ψ depends on Q itself, we show the following theorem.

Gallagher type theorem *For any ψ,*

$$\left|f - \frac{P}{Q}\right| < \frac{\psi(Q)}{|Q|}$$

has infinitely many solutions for a.e. f or at most finitely many solutions for a.e. f.

Then by using this theorem, we give a sufficient condition for the existence of infinitely many solutions, which we call Duffin-Schaeffer condition.

Duffin-Schaeffer condition *There are infinitely many positive integers n such that $\sum_{\substack{\deg Q \leq n \\ Q:monic}} \psi(Q) < C \sum_{\substack{\deg Q \leq n \\ Q:monic}} \psi(Q)\frac{\Phi(Q)}{|Q|}$ holds for a constant C, where ψ be a $\{q^{-n} : n \geq 0\} \cup \{0\}$-valued function which satisfies $\sum_{n=1}^{\infty} \sum_{\substack{\deg Q = n \\ Q:monic}} \psi(Q) = \infty$.*

In Chapter 3, we extend this problem to the multi-dimensional version, that is, the simultaneous approximation problem. In this case, we have a Duffin-Schaeffer type sufficient condition for having infinitely many solutions.

In Chapter 4, we have simultaneous approximate fractions by Jacobi-Perron algorithm, which is one of multi-dimensional continued fraction expansions. We give an estimate of the convergent rate of these fractions by the ergodicity of the map which induce the coefficients of Jacobi-Perron expansions.

Theorem *For any $\nu \geq 1$, we have*

$$|A_0^{(\nu)}| \left| f_i - \frac{A_i^{(\nu)}}{A_0^{(\nu)}} \right| \ll \frac{1}{|A_0^{(\nu)}|^{\frac{1}{r}(\frac{\gamma}{\rho} - \varepsilon)}} \qquad \forall \varepsilon > 0$$

for m^r-a.e. $(f_1, \ldots, f_r)$, where $\gamma = \frac{q^{r^2}}{q^{r^2}-1}$, $\rho = \frac{q}{q-1}$.

In Chapter 5, we discuss about modified Jacobi-Perron algorithm. We show the map, which induce the coefficients of its expansions, is ergodic and have a similar theorem as Jacobi-Perron algorithm.