THE SUMMARY OF Ph. D. DISSERTATION

No. 1

Major Mathematics		SURNAME, Firstname OGAWA, Masao

Title

Free Boundary Problems for an Incompressible Ideal Fluid

Abstract

We are concerned with the motion of the surface of an incompressible ideal fluid under the influence of gravity. The fluid occupies two-dimensional domain bounded by a rigid bottom and a free surface. This gives us a free boundary problem for an incompressible Euler equation. Such a free boundary problem has been often discussed under the assumption that the flow is irrotational. However, in this thesis, we are interested in the rotational flow. We study the unique existence of the solution, locally in time, in a class of functions of finite smoothness.

This thesis consists of four chapters.

In Chapter 1, we state the problem of water waves which we study and some known results for its well-posedness.

In Chapter 2, we study (1)the free boundary problem when the effect of the surface tension is negligible and gravity works downward. This problem is solved under the condition that the initial surface and the bottom are almost flat and the initial velocity is sufficiently small. Furthermore, we find that the time of the existence of the solution increases unboundedly as the initial data tend to zero.

In Chapter 3, we study (2)the problem with the effect of surface tension irrespective of the direction of gravity. The temporally local solution is established if similar conditions as above are satisfied. Moreover, it is shown that the solution of problem (2) converges to that of problem (1) as the coefficient of surface tension tends to zero.

In the last chapter, we study (3)the problem when the surface tension is not effective and the initial surface and the bottom are uneven. Then it is shown that this problem is well-posed. This result is a generalization of that in Chapter 2.

In order to show the unique solvability of the problem of water waves, we reduce the free boundary problem to the initial value problem on the surface and the boundary value problem in the interior. Solving these two problems by iteration, we obtain the solution of the original problem. In Chapters 2 and 3, some inverse operators in the problem on the surface are defined by the Neumann series. Hence, for the convergence of the series, we need the condition that the initial surface and the bottom are almost flat. However, in Chapter 4, we define the inverse operators by virtue of the potential theory. Then we can remove the restriction on the flatness of the boundaries.