THE SUMMARY OF Ph. D. DISSERTATION

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Title

Existence and Construction of Array Type Block Designs and Their Generalization to Edge-Colored Graph Decompositions

Abstract

In 1979, Singh and Dey introduced a *balanced incomplete block design with nested rows and columns* (BIBRC for short), which is posed from the statistical point of view. On the other hand, Raghavarao (1971) introduced a square lattice design. Recently, a square lattice design has been generalized to a *grid-block design* by Fu, Hwang, Jimbo, Mutoh and Shiue (2004) to utilize them for a pooling design in DNA library screening. BIBRCs and grid-block designs are classified into so-called "array type" designs, which are the main theme of this thesis.

For a finite set V with v elements, let $\mathcal{A}$ be a collection of b arrays of size $k_1 \times k_2$. Each array is called a *block*. We denote the numbers of blocks of $\mathcal{A}$ in which two distinct points x and y occur in the same row, in the same column and in the same block by $\lambda_R\{x, y\}$, $\lambda_C\{x, y\}$ and $\lambda_B\{x, y\}$, respectively. A pair $(V, \mathcal{A})$ is called a BIBRC if the following conditions are satisfied: (i) Every point occurs at most once in each block of $\mathcal{A}$. (ii) Every point occurs in exactly r blocks of $\mathcal{A}$. (iii) For any pair of distinct points x and y, $k_1\lambda_R\{x, y\} + k_2\lambda_C\{x, y\} - \lambda_B\{x, y\} = \lambda$ holds. Also, a pair $(V, \mathcal{A})$ is called a grid-block design if the conditions (i), (ii) and the following condition (iv) are satisfied instead of the condition (iii): (iv) For any pair of distinct points x and y, $\lambda_R\{x, y\} + \lambda_C\{x, y\} = 1$ holds.

In this thesis, we will discuss constructions of array type combinatorial designs and show the existence of such designs for some specific parameters.

In Chapter 1, we briefly describe some backgrounds of combinatorial designs studied in the succeeding chapters of this thesis.

In Chapter 2, we obtain two direct and one recursive constructions of grid-block designs by utilizing cyclic balanced incomplete block designs (cyclic BIB designs for short), affine geometries and group divisible designs. Further, the existence problem of grid-block designs is solved in the case of 2×4, 3×3 and $2 \times 2 \times 2$ grid-blocks for all integers satisfying the necessary conditions. The existence of 2×4 grid-block designs is shown by constructing nine grid-block designs and two group divisible grid-block designs with specific parameters and by applying the previous recursive construction to these grid-block designs. The existence of 3×3 grid-block designs is shown by utilizing the direct construction for cyclic BIB designs and constructing some other grid-block designs directly. Similarly, the existence of $2 \times 2 \times 2$ grid-block designs is shown by constructing cyclic or rotational grid-block designs directly. Besides, we discuss the constructions and existence of *resolvable* grid-block designs and packings.

In Chapter 3, we give constructions of BIBRCs by utilizing finite fields and affine geometries. Firstly, a construction of BIB designs is obtained by utilizing a partial set of t-flats of affine geometries. By applying this construction, constructions of nested BIB designs and BIBRCs are given. Next, a construction of BIBRCs having smaller λ than known other constructions is obtained by utilizing finite fields. Moreover, the existence of a BIBRC is shown when the number of points v is a sufficiently large prime power satisfying $\lambda(v-1) \equiv 0 \,(\mathrm{mod}\, k_1k_2(k_1-1)(k_2-1))$. And we give a table for the existence of BIBRCs with small parameters ($13 \leq v \leq 101$ and $3 \leq k_1 \leq k_2 \leq 11$) in Appendix A for practical use.

In order to show the existence of BIBRCs and grid-block designs for general array size $k_1 \times k_2$, it may be useful to generalize the notion of array type design to that of graph decomposition. Wilson (1976) showed an asymptotic existence theorem on the decompositions of complete graphs into simple graph. And Lamken and Wilson (2000) showed an asymptotic existence of simple edge-colored graph decompositions of complete graphs. By utilizing their method, we can claim asymptotic existence of grid-block designs. However, their method can not be applied to the existence of some kinds of array type designs like BIBRCs. In Chapter 4, we show asymptotic existence of *colorwise simple graph* decompositions of complete graphs by introducing a notion of *tree-ordered*.

In Chapter 5, the existence of BIBRCs is shown for sufficiently large integers satisfying the necessary conditions by utilizing the results in Chapter 4.