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Title

Boundary value problems and crack propagation in elastic or viscoelastic media with cracks

Abstract

We consider boundary value problems and crack propagation in (visco)elastic media with cracks. We study fracture phenomena that are supposed to occur in the case of earthquakes. We can arrive at the equations describing the (visco)elastic motion by the equilibrium of force, the first law of thermodynamics and the conservation of momentum. However, it is difficult to regard a crack as a free boundary, because it has never been succeeded to derive an equation describing a propagation of cracks.

In this thesis, we prove that the stationary problem in a domain with a crack has the unique solution in weighted Hölder class. Next, considering a quasi-stationary problem of virtual crack extension, we can treat crack propagation problems. And we consider a nonstationary problem with viscous effects in a domain including a fixed crack.

This thesis consists of four chapters.

In Chapter 1, we state a background and some known results which are concerned with Earthquakes and fracture mechanics.

In Chapter 2, we study (1) a boundary value problem for an infinite elastic strip with a semi-infinite crack. The equations of the displacement for a homogeneous isotropic elastic material consist of the constitutive law (Hooke's law) and the equilibrium conditions without any body forces in the state of plain strain. On the upper and lower boundaries of the strip Dirichlet and Neumann conditions are imposed, respectively. And on the crack we assume the free traction condition. Using the potential theory, we can prove that this problem has a unique solution in the weighted Hölder spaces by the Fredholm alternative.

In Chapter 3, we study (2) propagation of cracks under the same situation as (1). Then, we consider a quasi-stationary model of virtual crack propagation. Applying maximum energy release rate criterion, we can obtain an explicit expression of the direction of crack propagation dependent only on surface force.

In Chapter 4, we study (3) an initial-boundary value problem in an infinite viscoelastic strip domain with a semi-infinite fixed crack. By using the Laplace transform this problem is reduced to the problem for a system of elliptic partial differential equations. We can prove that this problem has a unique weak solution in Sobolev-type spaces by virtue of Riesz theorem and Parseval's equality.