THE SUMMARY OF Ph. D. DISSERTATION

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Title

Stochastic processes in random environments and their limiting processes characterized by the semi-selfsimilarity

Abstract

Selfsimilar stochastic processes are processes whose finite dimensional distributions are invariant under a suitable scaling of time and space. In the fields of statistical physics and mathematical finance, many applications of selfsimilar processes are studied. Semi-selfsimilarity is an extension of selfsimilarity and expected to offer higher flexibility to stochastic modeling of random phenomena because of its weaker scaling property.

In Chapter 3, we construct semi-selfsimilar diffusion processes on disconnected fractal sets on $\mathbf{R}$ as limits of suitably scaled random walks.

In Chapter 4, we consider homogenization problems on the sets discussed in Chapter 3. In the case of nested fractal sets, Kumagai and Kusuoka (1996) studied these problems and considered environments whose mean is finite. In our case, we can treat an environment whose mean is infinite and show the limiting process belongs to a new class of semi-selfsimilar processes.

In Chapter 5, we consider the limiting behavior of stochastic processes in random environments, which is described by a formal stochastic differential equation:

$$dX(t) = dB(t) - \frac{1}{2}W'(X(t))dt, \quad X(0) = 0, \tag{1}$$

where $\{B(t)\}$ is a one-dimensional Brownian motion and $\{W(x)\}$ is a semi-selfsimilar process which is independent of $\{B(t)\}$ and called an environment. We show that the semi-selfsimilarity of the environments implies that the limit distribution of a suitably scaled process $\{X(t)\}$ converges along a subsequence of $t \to \infty$. This result is an extension of the selfsimilar case studied by Kawazu, Tamura and Tanaka (1988).

In Chapter 6, we characterize the limit distributions obtained in Chapter 5 for some environments. We show that in case of a non-negative reflecting Lévy environment the limiting process is semi-selfsimilar with independent increments, and thus its marginal distributions are semi-selfdecomposable.

In Chapter 7, we study recurrence and transience problem of a diffusion process in a multi-dimensional environment. Such a process is composed of d independent copies of $\{X(t)\}$ corresponding to (1) for some environments.