THE SUMMARY OF Ph. D. DISSERTATION

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Title

Numerical Analysis of Thermo-electrically Conducting Fluids Using Vector Finite Element Method for Induction Equations

Abstract

It is well known that the vector finite element method is one of the powerful tools for solving electromagnetic problems. Using vector finite element method, the solenoidal condition for the magnetic flux density satisfies automatically in time increment. This is because the arrangement of variables, which are on the center of the face for the magnetic field and on the center of the edge for the electric field in an element. In this thesis, numerical scheme using vector finite element method for induction equations are developed to investigate magnetohydrodynamics. The followings are the composition of the present study and the knowledge for each chapter.

In chapter 1, background and purpose of the present thesis are summarized.

Chapter 2 is, at first, the introduction to the induction equations from the Maxwell equations. Second, vector shape functions are defined to show the facet shape functions and the edge shape functions. To achieve vector finite element formulation, induction equations are divided into two phases, magnetic field and electric field. Magnetic field is Faraday's law, which is formulated using facet vector finite element method. Electric field is the combination of the Ampere's law and Ohm's law, which is formulated using edge vector finite element method. Finally flows between parallel plates under constant magnetic field are numerically simulated to verify the present scheme.

In chapter 3, numerical analysis of thermo-electrically conducting fluids in a square cavity is carried out. Flow field and temperature field are analyzed using GSMAC finite element method. Induction equations are numerically analyzed with vector finite element method. Parameter study for Hartmann numbers are carried out and the computational results are in good agreement with B method. Total calculation time for B method is one point five time as fast as that for present method. The difference of calculation time is depends mainly on the calculation of the induction equations. It became obvious that calculation time for induction equations of the present method was quarter as fast as that of B method.

In chapter 4, the vector finite element method is applied to the 3D analysis. Numerical analysis of thermo-electrically conducting fluids in a cubic cavity is carried out. Using unequally divided mesh of $30 \times 30 \times 30$, verification of the present numerical scheme was carried out for Hartmann numbers. Computational results of the velocity vectors, temperature contours and the magnetic flux density vectors are compared with B method and both results are in good agreement with each other. Calculation speed of the present method is 30 % faster than the B method. Calculation time for induction equations is one third as fast as the B method. These computational results show the validity, stability and economy of the present results.

In chapter 5, to simulate flows in a differentially heated cavity with 8:1 aspect ratio, the GSMAC-FEM for Navier-Stokes equations, CIP-FEM for energy equation and vector finite element method for induction equations are applied. Flows with high Rayleigh numbers without magnetic field are investigated for Rayleigh number $Ra = 3.4 \times 10^5$ and find out the mesh dependency of the flows. Parameter study for Rayleigh numbers are carried out and the critical Rayleigh number with which the flow turns the steady oscillation to steady non-oscillation is estimated at 2.9×10^5. It is also clear that Nusselt numbers are in proportion to the quarter power of the Rayleigh numbers. Parameter study for Hartmann numbers under constant magnetic field is investigated to show the effect of the magnetic field to flow field and vice versa. Larger the magnitude of the variation for the flow field, larger the magnitude of the variation for the magnetic flux density. Large Hartmann number stabilizes the flow field because of its large Lorentz forces.

Chapter 6 is the conclusion of the thesis and describes a future view and a future subject.