SUMMARY OF Ph.D. DISSERTATION

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Title

Heavy Paths and Cycles in Weighted Graphs and Related Topics

Abstract

A weighted graph is one in which each edge is assigned a nonnegative real number, called the weight of the edge. A weighted graph with constant weight 1 has the same property as an unweighted graph, hence we can regard an unweighted graph as a weighted graph with special property, and conversely a weighted graph can be regarded as a graph whose weights of the edges are unbalanced. The aim of theoretical study on weighted graphs is to find out whether unweighted graphs and weighted graphs have the same property or not. And if there is some difference, then our interest turns to extra-conditions on weighted graphs to have the same property as unweighted graphs. Such studies are extended in this thesis. Furthermore the new concept, weighted Ramsey problem, is introduced.

The study on long cycles in graphs is now one of the main topic in graph theory. In particular, there are many results which deal with long cycles passing through some specified vertices, and the extension of this problem into weighted graphs has been also studied. In the previous results, the existence of heavy paths is used to prove the existence of heavy cycles passing through some specified vertices. However, in this thesis, we show that the existence of heavy fans in weighted graphs is very useful. Using this argument, we obtain some short proofs of the previous results, and also some new results, in which the number of specified vertices are enlarged, are shown. To prove the existence of heavy fan, new edge-contraction method is introduced, and this method is expected to play an important role in future research of this field.

Next, we study Bondy and Fan's theorem, and we show the existence of heavy cycles by adding a condition on the girth of a graph. Moreover, we observe the existence of heavy cycles with Fan-type condition and σ_k -type condition.

About heavy paths, we discuss with Dirac-type condition and Ore-type condition. The existence of heavy fans also plays a key role when we prove the existence of heavy paths passing through some specified vertices with Dirac-type condition.

Moreover, we deal with weighted Ramsey problem. Though there are a lot of works on Ramsey problem in unweighted graphs, the study of it in weighted case has not begun. So, we introduce a new concept of weighted Ramsey problem in this thesis.