SUMMARY OF Ph.D. DISSERTATION

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Title

Operator semi-selfsimilar processes and one of their constructions as limiting processes of random walks in random sceneries.

Abstract

   We study stochastic processes with operator semi-selfsimilarity. Examples of operator semi-selfsimilar processes are obtained as a limit process of random walks in random sceneries. We also give some theorems on operator semi-stable distributions for studying such stochastic processes.

   The first chapter is introduction and the next chapter contains known results and a study of random walks which belong to domain of partial attraction of operator semi-stable distributions. We show a limit theorem on the probability of returns to the origin for these random walks.

   In Chapter 3, we deal with some topics of operator semi-selfsimilar processes. If some time-scaled stochastic processes and space scaled by operator ones have all the same finite dimensional distributions, it is said to be operator semi-selfsimilar. At first, we show that space scaling operator is exponential represented by using sign operators. The representation form is slightly different from previous studies. We investigate the class of space scaling parameters which is not concerned in prior studies. We obtain the representation for operator semi-stable Lévy process indicating the property of the class.

   In Chapter 4, we define stochastic integrals with respect to the random measure induced by operator semi-stable Lévy processes. The integrated process has operator semi-selfsimilarity with stationary increment. We investigate the case integrand is local time of another 1-dimensional semi-stable Lévy process. The integrated process has the operator semi-selfsimilarity induced by both Lévy processes. We show the relations among the semi-selfsimilarities of integrated process, integrand Lévy process and integrator Lévy process.

   In Chapter 5, we study scaled limit processes of random walks in random sceneries in a similar way to Kesten and Spitzer's study. Random walks in random sceneries take same steps many times and therefore have strongly dependent increments. Their scaling is determined by both of random walks and random sceneries. And this scaling limit converges weakly in some function space to the integrated process whose integrand is a local time of one dimensional semi-stable Lévy process and integrator is operator semi-stable Lévy process defined in Chapter 4. To prove the convergence, the local limit theorem proved in Chapter 2 has an important role. This limiting process is turned out to be an operator semi-selfsimilar process with stationary increments by the discussion of the relation of semi-selfsimilarity in Chapter 4. In this thesis relative to prior works, we extend not only random sceneries but also random walks to semi-stable case. Thus, some condition for convergence is necessary in random walks and random sceneries. This condition is indicated by the discussion of stochastic integral in Chapter 4. Conclusively, we construct an example of stochastic integrated process defined in chapter 4 by the limiting processes of appropriate scaled partial sums of some random walks in random sceneries. This is also the example of operator semi-selfsimilar processes.