SUMMARY OF Ph.D. DISSERTATION

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Title

Campanato theory for the Rothe approximate system of parabolic partial differential equations

Abstract

We develop Campanato theory for the Rothe approximate system of parabolic partial differential equations, The Rothe approximate system is an elliptic-parabolic difference-partial differential system obtained by the difference analogue of the parabolic systems with respect to time. This method guarantees the existence of the approximate solution for the initial-boundary value problem of parabolic systems from the theory for elliptic partial differential equations, and none of our estimates are a priori. Campanato theory is introduced by S. Campanato, and Campanato-type estimates have played a fundamental role in the regularity analyses for variational problems and partial differential equations of elliptic and parabolic types. However, for the Rothe approximate systems, only the interior estimates are obtained in the case where the coefficients are constant. In this paper, we show that Campanato estimates for the Rothe approximate systems with constant coefficients hold near the boundary as well. By use of these estimates, we derive Campanato estimates for the Rothe approximate systems with variable coefficients. Furthermore, the application of Campanato estimates is also considered.

Chapter 1 is an introduction, which refers to the background, the aim, and the main results of our study.

In Chapter 2, we establish Campanato estimates for solutions of the Rothe approximate system of parabolic partial differential equations (Rothe approximate solution). Different from the case of parabolic systems, two cases occurs depending on the relation between the size of the region for estimation and the width of the time mesh. In either case, we show that Camapanto estimate of parabolic type hold independently of the approximation.

In Chapter 3, we apply the Campanato estimates in Chapter 2 to derive the Hölder continuity of the Rothe approximate solution and its spatial first derivatives. First we estimate the solution in the whole space-time domain, prove that the solution belongs to a certain discrete Morrey space, and estimate its discrete Morrey norm independently of the approximate solution. In estimation on the ordinary Morrey space, the center of the neighborhood has to move on the whole domain, but it suffices to let the points run discretely with respect to time due to the special inclusion relation for the Rothe approximate solution. We show that, as the width of the time mesh tends to zero, the Rothe approximate solution converges to the solution of the original parabolic systems with Hölder continuous spatial first derivatives.