SUMMARY OF Ph.D. DISSERTATION

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Title

Cycles Containing Specified Vertices and Edges and Trees with Bounded Degree in Graphs

Abstract

A cycle containing all the vertices of a graph is called a Hamilton cycle of the graph. A Hamilton path is a path containing all the vertices of the graph. Research on Hamilton cycles and Hamilton paths is one of the most studied topics in graph theory, and many extensions of them have been considered. In this thesis, we consider trees and cycles containing specified vertices and edges as one of those.

A partition of a graph is to partition a graph into some vertex-disjoint subgraphs. Under this thought, we can consider a Hamilton cycle as a partition into one cycle. Ore's sufficient condition for a graph to have a Hamilton cycle is well-known. Later, Brandt et al. indicated that the same condition implies that not only the existence of a Hamilton cycle but also the existence of a partition into the specified number of cycles. After that, partitions into cycles containing previously specified vertices or edges were considered. In this thesis, we combine these two cases, that is, we consider the case when both vertices and edges are specified. We give a minimum degree condition and a degree sum condition for the existence of such structure which are best possible. As related topics, we consider degree conditions for a graph or a bipartite graph to have short cycles containing specified edges.

A tree which has the same vertex set of a graph is called a spanning tree of the graph. A Hamilton path is a spanning tree with the property that each of its vertices has degree at most two. Under this consideration, research on spanning trees with bounded maximum degree has been done. In this thesis, we consider the existence of such a spanning tree which contains specified vertices as its leaves. We give a degree sum condition and an independence number condition for it.

Paths and cycles containing all the prescribed vertices were considered as extensions of Hamilton cycles and paths. On the other hand, this concept has not been applied to a tree so much. In this thesis, we consider two types of trees with bounded degree. One is a degree bounded subtree containing all the specified vertices. The other is a spanning tree with restrictions on the degree of the specified vertices. Degree sum conditions for the existence of them are given, each of which is best possible.