SUMMARY OF Ph.D. DISSERTATION

School	Student Identification Number	SURNAME, First name
Science for Open and Environmental Systems		TANAKA, Masato

Title

Development of Incompressible Hyper-elastic Shell Elements using Assumed Strains and Their Application to Buckling Analysis of Thin-walled Tube under Bending

Abstract

The numerical implementation of nonlinear finite element procedures for flexible thin-walled structures using incompressible hyper-elastic shells is indispensable in the application of practical engineering and medical fields to predict and evaluate the behaviors of rubber-like shell structures and biological soft tissues. The investigation of a flexible thin-walled tube under bending is especially important in various branches of modern technology such as the forming limit of bent pipe, manufacturing of medical tubes, elastomer seals and numerical model for blood vessels. When a long thin-walled tube, such as a hydraulic hose, is subjected to a bending moment, its cross-section becomes distorted. Due to the reduction of the geometrical moment of inertia as its cross-section flattens, the stiffness of the tube decreases with the increase of load. With further moment, the tube collapses with a local kink in its longitudinal direction. The ovalization phenomenon of the cross-section in bending thin-walled tubes was first investigated by Brazier. However, since this work was based on the assumption of infinitesimal strain, his theory could not cope with the kink phenomenon which is shown physically. The pre/post buckling analysis of this kink instability of thin-walled tubes under bending is still an open issue due to its strong nonlinearity.

In order to conduct finite element analysis of the above flexible thin-walled tube, not solid elements but shell structure elements which can treat the constitutive law for rubber-like materials allowing large elastic strains are required in view of computational efficiency, modeling effort and accuracy. In recent years, these finite strain shell elements accounting for thickness change have been presented. The elements use additional degrees of freedom for thickness change and three dimensional constitutive relations. However, these additional degrees violate computational efficiency and simplicity of modeling which are primary advantages of shell structure elements, and then derive the complicated formulations. In order to overcome this difficulty, a lot of formulations for the large strain shell elements are still being developed.

The objective in this study is to present a new formulation for robust and efficient finite shell elements to investigate the instability problem which shows strong nonlinearity such as buckling analysis of flexible thin-walled tube under bending. Firstly, a high-accurate incompressible hyper-elastic shell element using two dimensional constitutive relations is developed. Then, the performance of the presented shell element is illustrated by its applications to several numerical examples including bending analysis of a thin-walled tube. Finally, the nonlinear mechanism of kink phenomenon of flexible thin-walled tube under bending is clarified.

The composition of this thesis and the contents in each chapter are shown as follows.

In chapter 1, the background and the purpose of the present study are described.

In chapter 2, definition of hyper-elasticity and derivation of the constitutive law are summarized as the fundamental review of this study.

In chapter 3, a finite strain shell element with its material restricted to hyper-elasticity is developed. This shell element is based on the MITC shell element developed by Bathe to perform locking-free behavior using assumed transverse shear strains. In addition to this MITC formulation, an assumed transverse normal strain is introduced to treat thickness change. In this formulation, the transverse normal strain is assumed to be uniform throughout the element, and evaluated at the middle surface using the incompressibility condition. Indeterminate pressure, which occurs in incompressible materials, is eliminated at the element level using the plane stress condition. The advantage of this technique is that well-conditioned tangent stiffness for a large strain shell element can be obtained without any additional variables.

In chapter 4, a numerical implementation of the solid element and shell element with the Ogden material model is presented. This formulation derives the efficient constitutive law of Ogden material using invariants of the right Cauchy-Green tensor in the extension form of the Mooney-Rivlin's law, without solving eigenvalue problems or coordinate system transformations. This chapter also validates the relationship between the present expression of the constitutive law and numerical stability.

In chapter 5, buckling analysis of thin-walled tube under bending is investigated using the present hyper-elastic shell elements. Firstly, in order to compare with Brazier's solution, the numerical analysis of a Hookean cylindrical tube is conducted using MITC4 shell elements that neglect thickness changes. From the numerical analyses, it is revealed that there are two critical points, a bifurcation point and a limit or turning point that causes the snap-back phenomenon before the limit point that Brazier estimated. The critical point where the bifurcation point coincides with the limit point is called the "hilltop branching point". Symmetrical kinks are observed along the primary path in the final, largely deformed, configuration, while only one kink is observed along the bifurcation path. Then, by using the developed large strain shell elements in which the Ogden material model is utilized, the pre/post buckling behavior of a rubber tube under bending with thickness changes is thoroughly investigated. It is revealed that there is hilltop branching point consisting of one turning point that causes the snap-back phenomenon and two bifurcation points. The results of Ogden material model also shows two symmetrical kinks in the final deformed configuration, which are located on the inside of the tube relative to the results of the Hookean model.

In chapter 6, the concluding remarks of this thesis and the future works are given.