SUMMARY OF Ph.D. DISSERTATION

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Title

On a Construction of Characteristic Classes
for Bundle Gerbes from the Viewpoint of Algebraic Topology

Abstract

In this dissertation we shall discuss bundle gerbes and a construction of their characteristic classes. Also we will discuss the instanton Floer homology group for homology handles related to Seifert homology spheres as a related topic.

The primary subject is a geometric object called a bundle gerbe. This is a geometric realization of 3-dimensional integral cohomology. It is well known that 1- and 2-dimensional integral cohomology are identified with the homotopy classes of maps from the manifold to a circle S^1 and the isomorphism classes of complex line bundles, respectively. A bundle gerbe is considered as a higher analogy of a complex line bundle. In fact, a bundle gerbe behaves in a similar way to it.

Historically the geometric realization of 3-dimensional integral cohomology originated in the formulation of gerbes by Giraud who researched non-abelian cohomology. After that, Brylinski investigated the geometric properties of gerbes and exploit them for the central extension of loop groups. Murray invented the notion of bundle gerbes, which are more concrete objects than gerbes.

In Chapter 1 we explain the formulation of bundle gerbes and the Hitchin gerbes. Also we give the lifting bundle gerbe associated with a principal bundle and a central extension of the structure group.

In Chapter 2 we discuss the connection and curving on a bundle gerbe and the geometric properties. Moreover we give an exposition of the construction of bundle gerbes coming from geometry.

In Chapter 3 we introduce the notion of bundle gerbe modules. While a bundle gerbe is a higher analogy of a complex line bundle, a bundle gerbe module is that of a complex vector bundle. Moreover, the Grothendieck group of the bundle gerbe modules is isomorphic to the twisted K-group which is originated with J. Rosenberg. He generalized the K-group in a point of view of the family of Fredholm operators. When we try to describe the twisted K-group by vector bundle-like objects, the bundle gerbe modules appear. We explain the Chern-Weil construction of the twisted Chern character of bundle gerbe modules due to Bouwknegt-Carey-Mathai-Murray-Stevenson.

The main results are in Chapter 4. We introduce the notion of an n-trivialization and a compatible curving and construct the splitting of bundle gerbe modules to define the twisted Chern classes and the twisted Chern character for bundle gerbe modules in terms of algebraic topology. Moreover, we prove that the latter coincides with the twisted Chern character due to Bouwknegt-Carey-Mathai-Murray-Stevenson if the bundle gerbe is given an n-trivialization and a compatible curving.

In Chapter 5 we discuss the Dirac operators associated with the spin bundle gerbe modules for a possibly non-spin^c manifold.

In Chapter 6 we investigate the instanton Floer homology group of homology handles obtained by 0-surgery along a singular fiber in a Seifert homology sphere. The Floer homology groups appear as a kind of homology groups associated to a 3-manifold. Following the Morse theoretic description of homology of a finite dimensional manifold, Floer extracted the homology groups from the space of connections, which has infinite dimension, and the Chern-Simons functional.

We clarify a relationship among the Floer homology groups of homology handles obtained by 0-surgery along a singular fiber of a Seifert homology sphere.