SUMMARY OF Ph.D. DISSERTATION

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Title

Characterizations of Some Subclasses of Infinitely Divisible

Distributions on R^d by Stochastic Integrals

Abstract

A probability measure μ on R^d is called infinitely divisible if, for any positive integer n, there exist a probability measure μ_n on R^d such that $\mu = \mu_n^{n*}$, where μ^{n*} is the n-th convolution of μ. The class of infinitely divisible distributions is known as the most important class of probability distributions.

Historically, the results on classifying its subclasses were mainly given in terms of Lévy measure ν in the Lévy-Khintchine representation of the characteristic function. The characteristic function is the the Fourie transform of a probability measure. Hence, these results were analytical ones. Recently, probabilistic interpretations for such results have been interested in, and, especially, characterizations of subclasses of them by stochastic integrals with respect to Lévy processes have been well studied.

When we represent a Lévy measure ν by a polar decomposition, the properties of infinitely divisible distributions can often be written in terms of its radial component ν_ξ. A symmetric probability measure μ on R^d is said to be of type G if its radial component ν_ξ is represented as $\nu_\xi(dr) = g_\xi(r^2)dr$ with a completely monotone function $g_\xi(r)$. A function on $(0,\infty)$ is called completely monotone if $g(r) \geqq 0$ and $(-1)^n g^{(n)}(r) \geqq 0$. Here $g^{(n)}(r)$ is n-th order derivative of $g(r)$.

In this thesis, we characterize the class of type G distributions and their nested subclasses by stochastic integrals with respect to Lévy processes. Since there were no analytical results for nested subclasses of type G distributions, we also characterized them analytically.

On the other hand, a probability measure μ on R^d is called selfdecomposable if its radial component ν_ξ is represented as $\nu_\xi(dr) = r^{-1}k_\xi(r)dr$. To understand type G and selfdecomposable distributions more deeply, we defined a new class M as follows. A symmetric probability measure μ on R^d is in class M if its radial component ν_ξ is represented as $\nu_\xi(dr) = r^{-1} g_\xi(r^2)dr$. Distributions of the class M belong to both classes of type G and selfdecomposable distributions by its definition. Namely, the class M is a new subclass in the class of type G and selfdecomposable. We constructed nested subclasses of M and characterized the classes of M as well as its subclasses by stochastic integrals with respect to Lévy processes. We also characterized them in terms of Lévy measures. Furthermore, we showed some relations between the new class M and other well-known subclasses of infinitely divisible distributions. Some strict inclusions among classes were proved by some new examples of infinitely divisible distributions.