SUMMARY OF Ph.D. DISSERTATION

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Title

Realizations of fixed point subgroups by the automorphisms of finite order in exceptional simple Lie groups and its applications

Abstract

It is well known that the involutive automorphisms of the compact Lie groups play an important role in the theory of symmetric spaces (c.f. Berger [1]). In [42], [43] and [44] Yokota showed that the exceptional symmetric spaces G/H are realized definitely by calculating the fixed pont subgroups of the involutive automorphisms. J. A. Wolf and A. Gray classified the automorphisms of order 3 in the connected compact Lie groups of centerfree and the structure of the fixed point subgroups by its automorphisms([56]). Yokota realized the automorphsms of order 3 in exceptional compact Lie groups G_2, F_4 and E_6, and determined the structure of the fixed point subgroups by its automorphisms([41]).

For the case of the graded Lie algebra, the following results are investigated. Kaneyuki classified the second kind graded decompositions of simple Lie algebras :$\mathfrak{g} = \mathfrak{g}_{-2} \oplus \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2$ and the subalgebras $\mathfrak{g}_{ev} = \mathfrak{g}_{-2} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_2, \mathfrak{g}_0$([18]). Hara classified the third kind graded decompositions of simple Lie algebras :$\mathfrak{g} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_{-2} \oplus \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3$ and the subalgebras $\mathfrak{g}_{ev}, \mathfrak{g}_0$ and $\mathfrak{g}_{ed} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_3$([10]).

In this paper we shall realize those results by Lie group when the Lie algebra $\mathfrak{g}$ is exceptional type. For the purpose we consider the problem realizing fixed point subgroups by automorphisms of finite order and also the intersections of those fixed point subgroups. For the case of exceptional Lie group G, the involutive automorphisms $\sigma, \sigma' \in F_4,\ \gamma, \gamma' \in G_2$ play an important role. Here we determine the group structures of the intersection of the fixed point subgroups $G^\sigma \cap G^{\sigma'}, G^\gamma \cap G^{\gamma'}, G^\sigma \cap G^\gamma, G^\sigma \cap G^{\sigma'} \cap G^\gamma \cap G^{\gamma'}$. For the exceptional compact Lie group E_7, we determine the automorphisms of order 3 and the structure of the fixed point subgroups by its automorphism. There exists the spinor groups sequence of exceptional compact Lie groups: $Spin(1) \subset \cdots \subset Spin(8) \subset \cdots \subset Spin(14) \subset Spin(15) \subset Ss(16) \subset E_8$. Then we can prove that the sequence is deeply related to the fixed point subgroup of each spinor group by σ'. As the applications related to the automorphism of finite order, we consider the group realizations G_{ev}, G_0, G_{ed} which correspond to the Lie algebras $\mathfrak{g}_{ev}, \mathfrak{g}_0, \mathfrak{g}_{ed}$ investigated by Kaneyuki and Hara mentioned as above. Then we construct definitely the subgroup $Spin(14, C)$ of the complex exceptional Lie group ${E_8}^C$ and the structure of the group $({E_8}^C)_0$ in case of second kind graded decomposition. We also construct $Spin(12, C)$ of the complex exceptional Lie group ${E_7}^C$ and the structure of the group $({E_7}^C)_{ev}$ in case of third kind graded decomposition.

Remark. [1], [10], [18], [41], [42], [43], [44], [56] are the numbers of the references of the dessertation.