SUMMARY OF Ph.D. DISSERTATION

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Title		

Statistical Models for Data Which Include Angular Observations

Abstract

In a variety of scientific fields, observations are described as angles. On numerous occasions, these angular observations are jointly recorded with some other measurements such as linear and/or angular variables. Although various models have been proposed to analyze single angular data, comparatively little work has been done to model multiple observations which include angular ones.

This thesis provides some statistical models for dependence of observations which include angular ones. First we propose a regression model in which both covariates and responses are angular variables. The regression curve is expressed as a form of the Möbius circle transformation. The angular error is assumed to follow a wrapped Cauchy distribution. Some properties of the regression, including parameter estimation and testing procedures, are obtained.

Next we discuss some probability distributions for dependence of observations which include directional ones. We provide a theorem which constructs four-dimensional distributions with specified bivariate marginals on certain manifolds such as two tori, cylinders or discs. Some properties of the submodel of the proposed models are investigated. The theorem is also applicable to the construction of a related Markov process, models for incomplete observations, and distributions with specified marginals on the disc. Then two maximum entropy distributions on the cylinder are discussed. The circular marginal of each model is distributed as the generalized von Mises distribution which can be an asymmetric shape.

Finally we propose a bivariate model for a pair of dependent unit vectors which is generated by Brownian motion. Both marginals have uniform distributions on the sphere, while the conditionals follow the exit distributions. Some properties of the proposed model are investigated. Further study is given to the bivariate circular case by transforming variables and parameters into the form of complex numbers. Some desirable properties, such as multiplicative property and log-infinite divisibility, hold for this submodel. As a related topic, the proposed distribution is generalized so that both marginals have exit distributions.