SUMMARY OF Ph.D. DISSERTATION

School Fundamental Science and Technology	Student Identification Number	SURNAME, First name UMEHARA, Morimichi

Title

Global Solvability of the Free-Boundary Problem for Stellar Models of Self-Gravitating Viscous Radiative and Reactive Gas

Abstract

It is said that stars originate in processes of gathering many gaseous particles under their self-gravitations. However, when we observe the universe in the more macroscopic scale, the stars are gathered again by their own self-gravitations, and then, the large-scale structures of the universe, galaxies, galactic systems, etc., are constructed similarly. In the universe the self-gravitation is the uniform force constructing these several structures, and it is said that the set of moving media is regarded as the continuum in each space scale, for example, the set of stars in the stage where a galaxy is constructed.

In this thesis, keeping in mind the analysis of these astrophysical phenomena including the one in the stellar interior, we consider the free-problem for compressible, viscous and heat-conductive gas. In particular, we analyze the following two models: the three-dimensional spherically symmetric model with the central rigid sphere, and the one-dimensional model. For each model we take into account the following three qualities: (i) the self-gravitation of gas, (ii) the radiation phenomena, (iii) the self-producing energy and the variety of chemical composition of the media by chemical reactions. These are the essential phenomena in the astrophysical frameworks. The motion of gaseous stars is described by the system of partial differential equations corresponding to the conservation laws of mass, momentum and energy, and the equations of reaction-diffusion type determining the rate of mass of the reactants and the products. As an influence of radiation we take the Stefan-Boltzmann's law, which is known in the theory of radiation. Simultaneously, we take into account the influence of the energy transfer contributed by radiation. This is expressed in the thermal conductivity dependent nonlinearly on both the density and the absolute temperature. We also assume the first-order kinetics and take the Arrhenius law for the form of the reaction rate function. For these free-boundary problems, formulating the qualities mentioned above and based on known classical fundamental theorems of temporally local existence of unique solution, we obtain the suitable a priori estimates of the solution, and then, prove the temporally global existence of the unique classical solution.

This thesis consists of four chapters. In Chapter 1, we mention the physical background of our problems and historical studies of mathematical analyses for compressible, viscous (and heat-conductive) fluid. In Chapter 2 we formulate our two models mathematically and mention several known results closely related with our problems. In Chapter 3, we consider three-dimensional spherically symmetric model with the central rigid sphere. For our free-boundary problem formulated in Eulerian coordinate system, we transform that into the one with fixed domain by the Lagrangian mass transformation. Combining the representation formula of the density established first by Kazhikhov and Shelukhin, with L^2 energy estimates, we obtain the a priori estimates of the solution in Sobolev spaces. Furthermore, applying the Sobolev's imbedding theorem and the Schauder estimates, it is proven that unique solution of the problem exists in Hölder spaces globally in time. By using similar manner, in Chapter 4, we prove that the one-dimensional problem with general self-gravitation is also uniquely solvable globally in time.