SUMMARY OF Ph.D. DISSERTATION

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Title

Dynamic Properties of the Negative Slope Algorithm Arising from 3-Interval Exchange Transformations

Abstract

The negative slope algorithm (n.s.a.) is a 2-dimensional map defined on the unit square. This notion of the n.s.a. was introduced by S. Ferenczi, C. Holton and L. Zamboni in their study of the three letter languages arising from 3-interval exchange transformations.

In §1, we provide a brief history of interval exchange transformations and describe how the n.s.a. is deduced from 3-interval exchange transformations. In §2, we give the definitions of the n.s.a. and some basic notions related to the n.s.a. Then, in §3, we explain some sufficient conditions by the results of M. Yuri (1986) for multi-dimensional maps to be weak Bernoulli. In §4, we prove Rohlin's entropy formula and the weak Bernoulli properties of the n.s.a. by the results stated in §3. In §5, we construct a 4-dimensional map, which is called the natural extension of the n.s.a. and deduce the absolutely continuous invariant measure for the n.s.a. as the marginal distribution of the invariant measure for the natural extension of the n.s.a. We also calculate the explicit value of the entropy of the n.s.a. with respect to the absolutely invariant probability measure by Rohlin's entropy formula. In §6, we show the properties of the representation matrices of the n.s.a. and give a necessary condition for a point in the unit square stopping after finite steps. In §7, we show a necessary and sufficient condition for a point in the unit square being purely periodic under the n.s.a. as the main theorem of this section. As a corollary of this theorem, we give the characterization of eventually periodic points of the n.s.a. which S. Ferenczi, C. Holton and L. Zamboni showed. S. Ferenczi, C. Holton and L. Zamboni also introduced a slightly different algorithm deduced from 3-interval exchange transformations. We call this algorithm "the modified n.s.a." In §8, we give the definition of the modified n.s.a. and its basic notions. Then, in §9, we show some properties of the modified n.s.a. and prove Rohlin's entropy formula and weak Bernoulli properties of the modified n.s.a. as in §4. In §10, we construct a 4-dimensional natural extension map of the modified n.s.a. and deduce the absolutely continuous invariant measure for the n.s.a. as the marginal distribution. We also calculate the entropy of the modified n.s.a. by Rohlin's entropy formula. Finally, in §11, we characterize purely periodic points of the modified n.s.a. by using natural extension method.