SUMMARY OF Ph.D. DISSERTATION

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Title

Hamilton Cycles, Paths and Spanning Trees in a Graph

Abstract

A cycle in a graph is called a *hamilton cycle* if it passes all vertices of the graph. A problem of determining whether a given graph has a hamilton cycle or not is important in Graph Theory, but it is known as a "difficult" one in a Combinatorial sense. Therefore we do not deal with this problem directly, and consider from the following two aspects; to find better sufficient conditions for the existence of a hamilton cycle, and to study relaxed structures of a hamilton cycle. In this thesis, we focus on the second aspect, in particular, we consider degree sum conditions and independence number conditions for the existence of such structures.

A hamilton cycle must pass all vertices of a graph. Relaxing this property of a hamilton cycle, we consider a cycle passing all specified vertices. We shall give a sufficient condition for the existence of such a cycle in terms of degrees and independent sets of specified vertices. Moreover, a cycle passing not only specified vertices but also specified edges has been studied. We discuss about these cycles in Chapters 3 and 5, respectively.

As another notion of relaxing a hamilton cycle, we consider *a dominating cycle*. A cycle is called dominating if removing all vertices of it results in a graph with no edges. Definitely, a hamilton cycle is dominating, but the converse does not generally hold. We sometimes consider a dominating cycle as "close" to a hamilton cycle, because the outside of the cycle must be small. Moreover, it is known that a dominating cycle has some good properties as like a hamilton cycle. Therefore many researchers have studied a dominating cycle. We focus on a dominating cycle in Chapter 4. In Chapter 6, we introduce an invariant "Relative Length", which concerns with a property of a dominating cycle, and we mention the relationship between a dominating cycle or "Relative Length" and the length of a longest cycle of a graph in Chapter 7.

A *hamilton path* of a graph is a path passing all vertices. Similarly to a hamilton cycle, it is a difficult problem to determine whether a given graph has a hamilton path or not. We can also consider some relaxed concept of a hamilton path, like a hamilton cycle. In particular, spanning trees with particular properties have been much studied. For example, a spanning tree with bounded maximum degree, or with bounded number of vertices of degrees one, or with bounded number of vertices of degrees at least three. We focus on these spanning trees in Chapters 8–10, 11 and 12, respectively.