SUMMARY OF Ph.D. DISSERTATION

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Title

Transformations and linkages in triangulations on surfaces

Abstract

A triangulation on a surface is a simple graph embedded on the surface such that each face is bounded by a triangle. Triangulations are one of the most important objects in topological graph theory. In this thesis, we focus on two subjects of triangulations, one is the transformation of triangulations, and the other is the linkage problem in graphs on surfaces.

Let G be a triangulation on a surface and let e be an edge of G. A *diagonal flip* of e is to replace e with the other diagonal in the quadrilateral region formed by two faces incident to e. If this breaks the simplicity of the graph, then we do not apply it. In 1936, Wagner proved that any two triangulations on the sphere with the same number of vertices can be transformed into each other by diagonal flips. After that, Komuro proved that $8n–48$ diagonal flips are sufficient for two triangulations on the sphere with n vertices, though Wagner's proof gives an algorithm to transform one into the other by $O(n^2)$ diagonal flips. In Chapter 2, we prove that the number can be decreased to 6n–30, focusing on a Hamilton cycle in triangulation on the sphere. In Chapter 3, we have enhanced this result to the projective plane. We show the number of diagonal flips to be $O(n)$, improving the earlier result in which $O(n^2)$ is sufficient, where n is the number of vertices of the triangulation.

Chen et al. introduced the notion "(m,n)-linkage" of a graph, which is derived from Graph Minor argument related to a graph linkage problem. A graph G is said to be (m,n)-*linked* if for any two disjoint subsets R, B of $V(G)$ with $|R| \leqq m$ and $|B| \leqq n$, G has two disjoint connected subgraphs containing R and B, respectively. In Chapter 4, we show the necessary and sufficient condition for a planar graph to be (3,3)-linked. In Chapter 5, we give a sufficient condition for a triangulation on a surface to be (k,k)-linked, for k=3,4,5. It is remarkable that this simplifies the proof of the theorem in Chapter 4 and extends the argument to nonspherical surfaces, focusing on a relation between cycles and cut-sets in triangulations.