SUMMARY OF Ph.D. DISSERTATION

School Fundamental Science and Technology	Student Identification Number	SURNAME, First name OTSUKI, Rei

Title

On the modular elements and the Euler systems for an elliptic curve

Abstract

The theory of elliptic curves is important in number theory, and many mathematicians have studied the Mordell-Weil groups, the Selmer groups, and the Tate-Shafarevich groups. For an elliptic curve defined over the rational number field Q, two systems of elements were defined for cyclotomic fields. One is the Euler system of the zeta elements in the cohomology groups defined by Kato. The other is the system of the modular elements in the group rings of the Galois groups defined by Mazur and Tate. Both systems are related to the Selmer groups and the p-adic L-function for the cyclotomic Z_p-extension, but the relation between the two elements had not been known.

The first result on the relation between the two systems was Kurihara's result when he studied the Selmer groups in the supersingular case. For an odd prime number p, he studied the relation between the zeta elements and the modular elements in the finite extension fields in the cyclotomic Z_p-extension of the rational number field Q, and showed that the two elements correspond through a map which has nice integrality. He used the above correspondence to determine the structure of the Selmer groups.

In this paper, we studied the relation between the zeta elements and the modular elements in more general cases, and as the main result of this paper, we constructed a map that corresponds the two elements and obtained some properties of the map. We named a system which satisfies the same formulas with those of the modular elements an admissible system. We defined a map for an elliptic curve E over Q and for arbitrary cyclotomic fields in the case where E has good reduction at p, and proved that an Euler system corresponds to an admissible system through the map, and in particular, the zeta element corresponds to the modular element. We also proved the integrality of the map for many cases. We can regard Kurihara's map as a special case of our map.

In this paper, we also obtained the similar result to Kurihara's in the case where $p = 2$. In this case, the corank of the Selmer groups is positive while the Selmer groups are finite in Kurihara's result.