For $\lambda = (\lambda_1, \ldots, \lambda_m)$ and $\alpha = (\alpha_1, \ldots, \alpha_m)$, we put $|\lambda| = \sum_{i=1}^{m} \lambda_i$ and $\alpha^\lambda = \prod_{i=1}^{m} \alpha_i^{\lambda_i}$. Let $r \geq 2$ be an integer. We define $\Omega_n z := (z_1^{n_1}, \ldots, z_m^{n_m})$ for $z = (z_1, \ldots, z_m)$ and consider the function

$$
\Phi_0(z) = \sum_{k \geq 0} \frac{E_k(\Omega_n z)}{F_k(\Omega_n z)} \in K[[z]] = K[[z_1, \ldots, z_m]],
$$

where $K$ is an algebraic number field and

$$
E_k(z) = \sum_{1 \leq |\lambda| \leq L} e_{k\lambda} z^\lambda, \quad F_k(z) = 1 + \sum_{1 \leq |\lambda| \leq L} f_{k\lambda} z^\lambda \in K[z]
$$

are coprime. We assume that $\log |e_{k\lambda}|, \log |f_{k\lambda}| = o(r^k)$. For an algebraic number $\alpha$, $|\alpha|$ is defined by $\max(|\alpha|, \text{den}(\alpha))$, where $|\alpha|$ and $\text{den}(\alpha)$ are the maximum of the absolute values of the conjugates of $\alpha$ and the least positive integer such that $\text{den}(\alpha)$ is an algebraic integer, respectively. The function $\Phi_0(z)$ satisfies a Mahler type functional equation.

The main theorems of the thesis are as follows:

**Theorem 1.** Let $\alpha = (\alpha_1, \ldots, \alpha_m) \in (K^\times)^m$ with $0 < |\alpha_1|, \ldots, |\alpha_m| < 1$ such that $F_k(\Omega_n \alpha) \neq 0$ for every $k \geq 0$. Assume that $|\alpha_1|, \ldots, |\alpha_m|$ are multiplicatively independent. Then $\Phi_0(\alpha)$ is algebraic if and only if $\Phi_0(z)$ is a rational function over $K$.

Theorem 1 insists the equivalence between the rationality of the Mahler function $\Phi_0(z)$ and the algebraicity of the value of the function at an algebraic point. Specializing Theorem 1, we get some transcendence results of reciprocal sums of binary linear recurrences. Let $\{R_n\}_{n \geq 0}$ be a binary linear recurrence satisfying

$$
R_{n+2} = AR_{n+1} + BR_n,
$$

where $A, B, R_0, R_1 \in \mathbb{Z}$ with $(A, B), (R_0, R_1) \neq (0, 0)$. Assume that $\Delta = A^2 + 4B$ is positive. Let $\sum_{k \geq 0} \frac{a_k}{R_k}$ be a sum taken over all $k \geq 0$ such that $R_k \neq 0$.

**Theorem 2.** Let $\{R_n\}_{n \geq 0}$ be a binary linear recurrence defined by (1). Suppose that $\{R_n\}_{n \geq 0}$ is non-periodic and $R_{r+k} \neq 0$ for infinitely many $k$. Let $\{a_k\}_{k \geq 0}$ be a sequence in $K$ such that $a_k \neq 0$ for infinitely many $k$ and $\log |a_k| = o(r^k)$. Then

$$
\theta = \sum_{k \geq 0} \frac{a_k}{R_k} \notin \mathbb{Q}
$$

except in the following two cases:

1) Let $r = 2$, $a_n = a (n \geq N)$ for some $a \in K$ and $N \in \mathbb{N}$, $|B| = 1$, and $R_0 = 0$. Then $\theta \in K(\sqrt{\Delta})$.

2) Let $r = 2$, $a_n = a2^n (n \geq N)$ for some $a \in K$ and $N \in \mathbb{N}$, $A = \pm(B - 1)$, and $AR_0 = 2R_1$. Then $\theta \in K$. 