Estimation of Optimal Portfolio Weights Using Shrinkage Technique

Abstract
The mean-variance optimization is one of the standard frameworks used to obtain optimal portfolio weights. This framework requires estimators for the mean vector and the covariance matrix of excess returns. The classical method is to adopt the usual sample estimates for the mean vector and the covariance matrix. However, it is well known that the optimal portfolio weights obtained by the classical approach are unstable and unreliable.

In order to reduce the estimation error of the estimated mean-variance optimal portfolio weights, some previous studies have proposed applying shrinkage estimators. However, only a few studies have addressed this problem analytically. Since the form of the loss function used in this problem is not the quadratic one used in statistical literature, there have been some difficulties in showing analytically the general dominance results.

In this Ph.D. dissertation, we show the dominance of a broader class of Stein type estimators for the mean-variance optimal portfolio weights, which shrink toward the origin, a fixed point, the grand mean, or more generally, toward a linear subspace when the covariance matrix is unknown and is estimated. Most of previous studies have addressed this problem when we have no constraint on portfolio weights. However, we also show the dominance when there are linear constraints on portfolio weights, similarly to Mori (2004), who has shown a result for that case.

The obtained results enable us to clarify the conditions for some previously proposed estimators in finance to have smaller risks than the classical estimator which we obtain by plugging in the sample estimates. Jorion’s (1986) estimator, Black and Litterman’s (1992) estimator and Kan and Zhou’s (2007) estimators have been considered. We also propose a new improved estimator which utilizes a prior information about Sharpe ratio, which is a well known performance measure of funds.