Forbidden Induced Subgraphs Implying Properties in Graphs

Gabriel M. Sueiro

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Thesis Abstract

Registration Number	□ "KOU" □ "OTSU" No. *Office use only	Name	Sueiro Gabriel Marcelo

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Thesis Summary

Let G be a connected graph. For another connected graph H, we say that G is H-free if G does not contain H as an induced subgraph. For a family $\mathcal{F}$ of connected graphs, we say that G is $\mathcal{F}$-free if G is H-free for every graph H in $\mathcal{F}$. In this case we say that $\mathcal{F}$ is forbidden in G.

Given a property P on graphs, we can ask the following question. What are the families of connected graphs $\mathcal{F}$ such that for every large enough $\mathcal{F}$-free graph G, G satisfies P. This problem has been studied before for several properties in particular. For example, hamiltonian graphs, traceable graphs, graphs containing a 2-factor, pancyclic graphs, hamilton-connected graphs, cycle extendable graphs, graphs containing a perfect matching.

Giving a full characterization of all families of forbidden subgraphs sometimes might be difficult. For example, in the cases of hamiltonian graphs and traceable graphs, only the families consisting of one, two or three graphs are known. Even though in general it might be a difficult problem, in this thesis we give a full characterization for several classes of graphs. In this thesis we analyze the problem for the following five classes of graphs: claw-free graphs, star-free graphs, graphs having a perfect matching, graphs having a near perfect matching, t-tough graphs.

For claw-free graphs, we give a complete characterization of all the families of connected graphs $\mathcal{F}$ such that every large enough $\mathcal{F}$-free graph is claw-free. We also generalize this result to star-free graphs. Concretely, given a positive integer t, we show all the families of connected graphs $\mathcal{F}$ such that every large enough $\mathcal{F}$-free graph is $K_{1,t}$-free.

For graphs with perfect matchings, all the forbidden families of graphs consisting of at most 3 graphs were showed in previous works. For graphs with near perfect matchings, all the forbidden families containing only triangle-free graphs were showed in previous works. For both classes, we complete the characterization by showing all the families of connected graphs $\mathcal{F}$ such that every large enough $\mathcal{F}$-free graph of even (odd) order has a perfect matching (near perfect matching).

Finally, for t-tough graphs, given a real number t, we show all the families of connected graphs $\mathcal{F}$ such that every large enough $\mathcal{F}$-free graph is t-tough.

Additionally, for claw-free graphs and graphs with a perfect matching, we show the families that one gets when adding the condition $|\mathcal{F}| \leq k$ for each positive integer k.