Damage Detection of Frame Structure Using Harmonic Excitation based on Uncertain Modeling and Measurements

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ABSTRACT:

A technique of structural damage detection was proposed which considers the effects of both measurement noise and modeling errors in the baseline finite element model. Damage that accompanies changes in structural parameters can be estimated for a damaged structure from the change between measured vibration responses and ones calculated from the analytical model of the intact structure. In practice, modeling errors exist in the analytical model due to material and geometric uncertainties and a reduction in the degrees of freedom as well as measurement errors, making identification difficult. To surmount these problems, bootstrap hypothesis testing, which enables statistical judgment without information about these errors, was introduced. The technique was validated by real vibration data for a three-story steel frame structure.

INTRODUCTION

Vibration-based damage identification methods are based on the fact that structural damage usually causes a reduction in structural stiffness which are accompanied by changes in vibration characteristics. In most methods, damages is estimated by comparing changes in vibration responses in the damaged state with analytical baseline findings for the finite element model in the intact state. The baseline model has to express the intact structure accurately, which it is impossible even though the initial model is updated to match the measured responses as closely as possible. Modeling as well as measurement errors always must be coped with.

The effects of both measurement and modeling errors therefore must be considered to obtain identification results. The research dealing with these effects is scant. Xia [1]

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studied the influence of these uncertainties and estimated the probability of damage existence. Stiffness parameter statistics were derived by the perturbation method, and the probabilistic distribution was determined from Monte Carlo simulation results. Yeo [2] also used the data perturbation method and obtained a statistical distribution of system parameters. They both assessed damage by a hypothesis test. The perturbation method assumes a normal distribution, but measurement distributions and modeling errors do not always fit a normal distribution. Moreover, the method to determine covariance matrices is not self-evident. In a real situation, we can not determine the distribution functions or the covariance matrices of measurement and modeling errors.

A method of structural damage identification is presented here that considers the effects of both measurement errors and modeling errors. Application of the bootstrap method [3], an approach for data resampling, which requires no assumption of distribution functions or covariance matrices, is introduced. Hypothesis testing by that method was conducted to deal with these uncertainties using only measurement data. When damage is detected, bootstrap hypothesis testing enables one to judge whether that damage is due to real damage or to measurement and modeling errors. The method was validated by the use of real vibration data for a three-story steel frame structure.

MODEL UPDATING TECHNIQUE USING FRF DATA CHANGES

The forced vibration response of an um-damped system generated by harmonic excitation is obtained by solving the equation of motion;

$$M\ddot{x}(t) + Kx(t) = f(t) \tag{1}$$

where M and K respectively are the mass and stiffness matrices of the baseline model, $x(t)$ the displacement response vector, and $f(t)$ the vector of the external harmonic excitation force.

The forced vibration response in the frequency domain is;

$$X(\omega) = H(\omega)F(\omega) \qquad H(\omega) = [-\omega^2 M + K]^{-1} \tag{2}$$

where $X(\omega)$ and $F(\omega)$ respectively are the Fourier transforms of $x(t)$ and $f(t)$, $H(\omega)$ is the transfer function of the baseline model, and ω is the excitation circular frequency.

When stiffness is changed by δK from the baseline model, the equation of motion becomes

$$M(\ddot{x}(t) + \delta\ddot{x}(t)) + (K - \delta K)(x(t) + \delta x(t)) = f(t) \tag{3}$$

where $\delta x(t)$ is the increase in the displacement response. Then δk_e denotes the proportional changes in the stiffness of the e-th element.

Variations in the total stiffness matrices therefore are expressed as sums of changes in element stiffness matrices;

$$\delta K = \sum_{e=1}^{n} \delta k_e K^e \tag{4}$$

When parameters are updated to approximate the intact structure, they are denoted by δk_e^i. Likewise, when parameters of the damaged structure are updated, they are denoted by δk_e^d.

The Fourier amplitude $X'(\omega)$ of the displacement response of the real structure is

$$X'(\omega) = X(\omega) + \delta X(\omega) = H(\omega)F(\omega) + \sum_{e=1}^{n} S^e(\omega)F(\omega)\delta k_e \tag{5}$$

where $S^e(\omega)$ is

$$S^e(\omega) = H(\omega)K^e H(\omega) \tag{6}$$

The frequency response function (FRF), obtained by dividing the Fourier amplitude of the acceleration response by the Fourier amplitude of the harmonic excitation force, is used. On the assumption that the excitation force is applied at point *j* and acceleration is measured at point *i*, the FRF $a(i, j, \omega)$ is

$$a(i,j,\omega) = -\omega^2 X'(\omega)/F(\omega) = -\omega^2 (H_{ij}(\omega) + \sum_{e=1}^{n} S_{ij}^e(\omega)\delta k_e) \quad (7)$$

Transposing the unknown terms to the left side and the known ones to the right side,

$$-\omega^2 \sum_{e=1}^{n} S_{ij}^e(\omega)\delta k_e = a(i,j,\omega) + \omega^2 H_{ij}(\omega) \quad (8)$$

In Eq.(8), the measurement point *i*, excitation point *j*, and excitation frequency ω are arbitrary values. Separating the complex parameters into real and imaginary parts, and choosing *l* different sets of *i, j,* ω, this relationship can be written as a set of simultaneous equations;

$$\begin{bmatrix} \boldsymbol{ReU} \\ \boldsymbol{ImU} \end{bmatrix} \{\delta \boldsymbol{k}\} = \begin{Bmatrix} \mathrm{Re}\,\boldsymbol{\delta a} \\ \mathrm{Im}\,\boldsymbol{\delta a} \end{Bmatrix} \quad (9)$$

Thus, *l* different FRF may give 2*l* equations for 2*n* unknowns. By solving Eq.(9), the structural parameters can be updated.

This model provides an updating technique for both intact and damaged structures, and identifies δk_e^i (intact structure) and δk_e^d (damaged structure). The model is updated such that the FRFs from updated model match the measured FRFs.

This is the deterministic model updating technique. If the same measurements are conducted many times, and different values are identified, they can be assumed to be random variables for statistical identification.

STATISTICAL PROCEDURE FOR DAMAGE ESTIMATION

Bootstrap method

The bootstrap method is a resampling technique formulated by Bradley Efron in 1979 [3]. It is a computer intensive method in applied statistics, and a type of Monte Carlo method based on observed data. It obtains the distribution of sample statistic by iterative random selection. Its advantages are that it can be applied to data which do not have a normal distribution, and it constructs all types of probabilistic density functions automatically by means of data resampling.

Hypothesis test based on the bootstrap method

Hypothesis testing is a statistical approach to judge whether measured data are contradictory to the assumed hypothesis. Two random variables $X = (x_1, \cdots, x_m)$ and $Y = (y_1, \cdots, y_m)$ are assumed with μ_x and μ_y as their respective means. *m* is the number of samples. A problem to test that the X equals Y is considered. The null hypothesis and an alternative hypothesis becomes

$$H_0 : \mu_x = \mu_y \qquad H_A : \mu_x > \mu_y \quad (10)$$

The statistical test uses a test statistic, *T*, that measures the discrepancy between the data and the null hypothesis;

$$T(X,Y) = (\overline{X} - \overline{Y}) / \sqrt{S_x{}^2 / (m-1) + S_y{}^2 / (m-1)} \quad (11)$$

where $\overline{X}$ and $\overline{Y}$ are sample means, and $S_x{}^2$ and $S_y{}^2$ sample variances;

$$\overline{X} = \Sigma_{i=1}^m x_i / m \qquad \overline{Y} = \Sigma_{i=1}^m y_i / m \qquad S_x{}^2 = \Sigma_{i=1}^m (x_i - \overline{X})^2 / m \qquad S_y{}^2 = \Sigma_{i=1}^m (y_i - \overline{Y})^2 / m \quad (12)$$

Let t_{obs} denote the observed test statistic value. Whether the observed value of the test statistic, t_{obs}, is large can be judged from the p-value;

$$p = \Pr\{T \geq t_{obs} \mid H_0\} \tag{13}$$

If the p-value is below the level of significance, α, the null hypothesis parameter value is outside of the confidence set. The distribution of $T(X,Y)$ must be calculated under the null hypothesis. Traditional statistical analysis assumes that the distribution is approximately independent and the distributions of X and Y are normal, but actually, these distributions are unknown. The bootstrap method does not require such assumptions. It automatically generates the distribution under the null hypothesis using only the measured data.

In this study the distribution of $T(X,Y)$ was obtained based by the position arrangement method [4]. First, the observed data, X and Y, are transformed, then pseud-data which are assumed to fit the null hypothesis are obtained;

$$x_i^* = x_i - \overline{X} + \overline{Z},\ i = 1,\cdots,m \qquad y_i^* = y_i - \overline{Y} + \overline{Z},\ i = 1,\cdots,m \qquad \overline{Z} = (\overline{X} + \overline{Y})/2 \tag{14}$$

where $\overline{Z}$ is the sample mean of all the variables belonging to X and Y.

The empirical distribution is obtained from pseud-data

$$F_m(x) = \frac{1}{m}\sum_{i=1}^{m}\delta(x_i^* \leq x) \qquad G_m(x) = \frac{1}{m}\sum_{i=1}^{m}\delta(y_i^* \leq y) \qquad \delta(S) = \begin{cases} 1 & if \quad the \ \ event \ \ S \ \ is \ \ true \\ 0 & otherwise \end{cases} \tag{15}$$

Here $\delta(\cdot)$ is the indicator function, where

Next, samples are generated randomly, $X^{*b} = \{x_i^{*b},\cdots,x_n^{*b}\}$ and $Y^{*b} = \{y_i^{*b},\cdots,y_n^{*b}\}$, from the distribution, $F_m(x)$ and $G_m(y)$, and the value of the bootstrap statistical test t^{*b} is calculated. By iterating the above procedure B times, the approximation of p-value is;

$$p = \Sigma_{b=1}^{B}\delta(t^{*b} \geq t_{obs})/B \tag{16}$$

where B is called the bootstrap replication number.

If $p > \alpha$, the null hypothesis is adopted, otherwise it is rejected. α is the level of significance that must be decided in advance. With a large α, the number of undamaged elements remaining as damage candidates becomes large, requiring more iterations. In contrast, the possibility of judging a damaged element as being undamaged becomes large with a small α. In this study 5% was chosen for α, as often is adopted in statistics.

Procedure to estimate damage existence

In this study, elements whose stiffness is reduced are modeled as damaged elements. The null hypothesis and the alternative hypothesis are

$$\delta k_e^d = \delta k_e^i \qquad \delta k_e^d > \delta k_e^i \tag{17}$$

If the null hypotheses are accepted, it indicates that the element is not damaged.

The procedure proposed in this study is:

Step1: Obtain *m* groups of measurement data.

Step2: Identify stiffness changes for each element *m* times using the *m* data groups.

Step3: Apply bootstrap hypothesis testing for each element.

Step 4. Check if the null hypothesis is accepted for stiffness. If yes, the element is identified as undamaged and excluded as a damage candidate, otherwise damage is still suspected.

Step 5. Check if any elements were excluded in Step 4. If yes, identify the stiffness changes for the remaining elements *m* times using the *m* data groups, then go back to step 4, otherwise stop here.

Bootstrap hypothesis testing detects undamaged elements. It iteratively zooms in on damaged elements by step by step excluding those elements assessed as undamaged from the damage candidates.

EXPERIMENTAL VERIFICATION

Description of the 3-story steel frame structure

The validity of the proposed method was tested based on field measurements for a 3-story steel frame structure constructed at the Disaster Prevention Research Institute, Kyoto University, Japan. The elevation view is shown in Fig. 1.

The structure's dimensions were 7m (width) × 9m (height) × 4m (depth), and each floor was 3m high. This structure was modeled with 2-dimensional beam elements. It had 6 columns, 3 floors, and 6 braces. All the members had rigid connections.

As for the columns, the section area and geometric moment of inertia respectively were 2.02×10^{-2} m^2 and 1.4313×10^{-4} m^4. As for the braces, respectively the values are 3.81×10^{-4}m^2 and 1.6028×10^{-7}cm^4. Both the columns and braces were made of steel which had a Young's modulus of 2.1×10^{11} N/m^2 and a mass density of 7.1 t/m^3. Two column members were combined in the longitudinal direction as one finite element to model the column. Similarly, to model the brace, two brace members were combined in the longitudinal direction as one finite element. The floors consisted of two steel beams and a reinforced concrete slab. The beams had a section area of 2.72×10^{-2} m^2, and a geometric moment of inertia of 2.098×10^{-3} m^4. The concrete slab had a section area of 1.2 m^2, a geometric moment of inertia of 0.09 m^4, a Young's modulus of 2.0×10^{10} N/m^2, and a mass density of 2.6t/m^3. To model the floor, the beams and concrete slab in the longitudinal direction were combined as one element.

Because a 2ton shaker was set at the center of the top floor (Fig. 1), a node was set at the position of the shaker. Therefore, the number of nodes is 9, and the number of elements 16. The number of elements assumed for identification, however, was 15 due to combining the two elements at the top floor on the presumption that they have the same properties. Numbering of the nodes and elements is shown in Fig. 2.

As this time only 6 measurements could be obtained, the structure was modeled with 15 elements; the minimum number that describe a structure with finite element modeling.

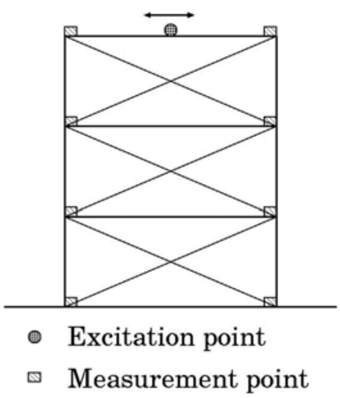


Fig.1 3-story frame structure

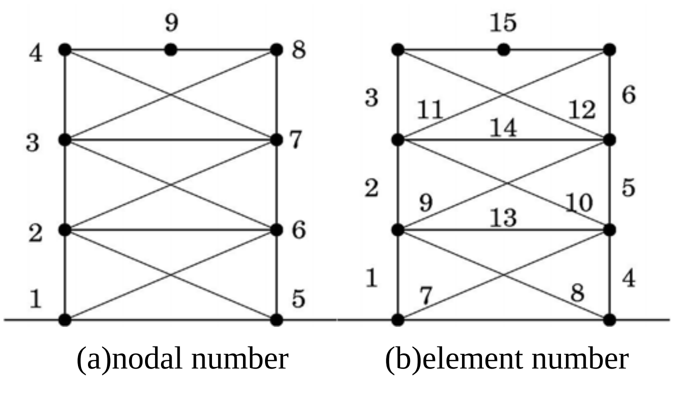


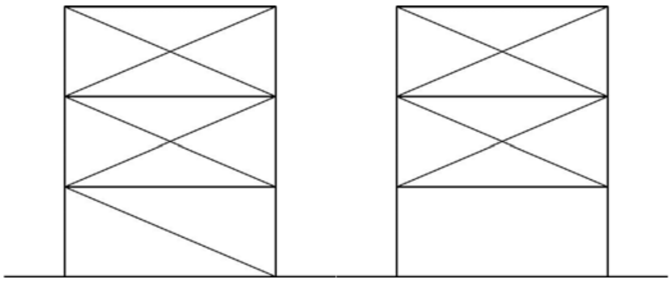
Fig.2 Analytical model of the 3-story frame structure

Damage models

Damage was modeled by extracting braces. Two damage models were assumed (Fig. 3). The first is a model without element No.7 (a brace at the first floor), meaning that the stiffness reduction of this element is 1.0 (100%). The second is the model without elements Nos. 7 and 8 (two braces at the first floor), meaning that the stiffness reduction of these elements is 1.0 (100%).

Experiment condition

Node No. 9 is the excitation point at which a 7.936kN harmonic excitation force is applied at the frequency of 2Hz. The excitation direction was horizontal, and acceleration responses in its direction measured at 6 nodes (Nos. 2, 3, 4, 6,7,8), provided 6 measurements for 15 elements (Fig.1). We measured 15 FRF combinations.



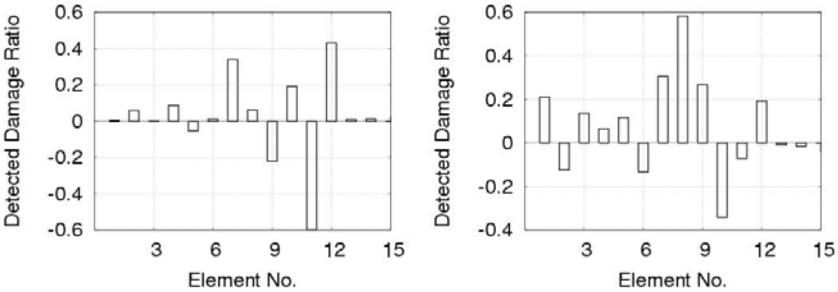
(a)damage model 1 (b)damage model 2

Fig.3 Damage model of the 3-story frame structure

Deterministic damage identification results

The baseline finite element model must be defined to identify damage. We created it from a draft of the structure. Because damping was very small, we modeled the structure with un-damped beam elements. Damage therefore is expressed only by the stiffness reduction.

Stiffness parameters were updated to match those of intact model, damage model 1, and damage model 2 based on the stiffness properties of the baseline model. Stiffness reduction due to damage then could be obtained. Identification results for the two damage models are shown in Fig. 4.



(a) damage model 1 (b) damage model 2

Fig. 4 Deterministic damage identification results

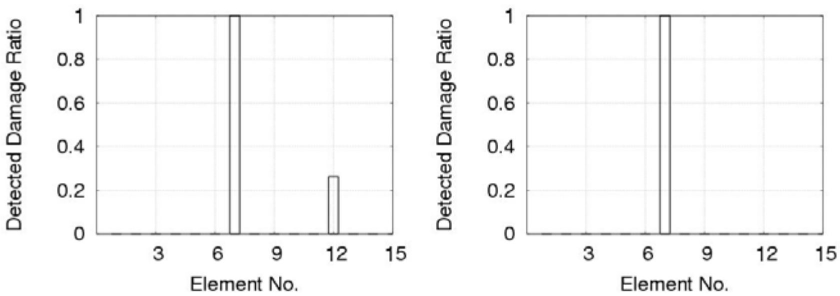
For damage model 1, the stiffness reduction identified for the 100% damaged element No.7 was only 38%. Moreover, the results show that undamaged elements Nos.10 and 12 have the possibility to be damaged and that the stiffnesses of elements Nos. 9 and 11 are greatly increased. This overestimation of elements No.9 and 11 is balanced by the underestimation of elements Nos. 10 and 12.

For damage model 2, the identified stiffness reductions for the two extracted braces are smaller than the actual values, and some undamaged elements are identified as damaged. The findings for the two cases are not good.

These results show the limitation of the deterministic damage identification technique. This is why we propose a statistical damage identification technique that prevents this type of identification failure by means of bootstrap hypothesis testing.

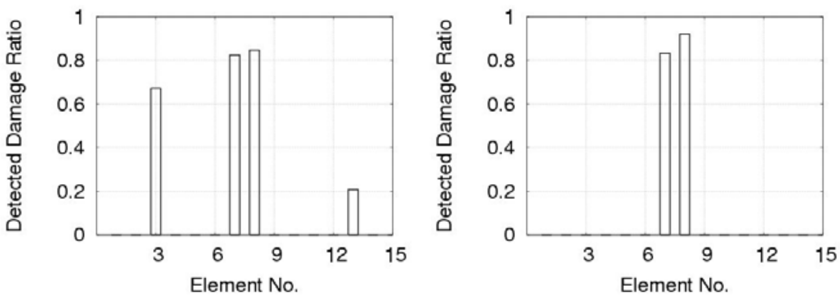
Statistical damage identification results

Hypothesis testing using the bootstrap method was conducted. In this case, the number of data, *m*, is 15, the bootstrap replication number, *B*, is 10000, and the level of significance is assumed to be 5%. Results are shown in Figs. 10 and 11. The vertical axis shows the bootstrap estimator of the mean values of δk_e of the elements for whom the null hypothesis was rejected.



(a) 1st calculation (b) 2nd calculation

Fig. 5 Statistical damage identification results; damage model 1



(a) 1st calculation (b) 2nd calculation

Fig. 6 Statistical damage identification results; damage model 2

For damage model 1, final results were obtained after 2 calculations. The first, narrowed suspicious elements to 2 elements, Nos. 7 and 12 (Fig.5(a)). The second, excluded element No. 12, the final result being that element No.7 was damaged 100% (Fig.5(b)), showing perfect accuracy.

For damage model 2, identification required 2 steps. In the first, suspicious elements were narrowed to 4; Nos. 3, 7, 8, and 12 (Fig.6(a)). In the second, two undamaged elements were excluded. The final results showed that element No.7 was about 84% damaged and element No.8 about 92% (Fig.6(b)). The stiffness reduction identified is not perfect, but the method succeeded in detecting the two damaged elements and in improving identification accuracy. The feasibility of bootstrap hypothesis testing also was validated by the experimental results.

CONCLUSION

A statistical damage identification technique for structures was presented that uses harmonic excitation force, which deals with uncertainty due to measurement noise and modeling errors in the baseline model. It combines the deterministic damage identification technique with hypothesis testing based on the bootstrap method. The deterministic damage identification method is derived from the fact that structural damage usually causes a reduction in structural stiffness, and these are accompanied by changes in vibration characteristics. The changes in vibration responses provide information about the locations and magnitudes of damage. This method detects damage perfectly for the ideal case of no measurement or modeling errors, but no such case actually exists. To deal with the effects of these errors, hypothesis testing using the bootstrap method was introduced. It enables statistical judgment of whether an element is damaged.

An experiment on a 3-story steel frame structure was conducted to verify the proposed technique. Damage was expressed by extracting braces. The proposed method correctly detected the damaged elements.

We have a plan to use a small portable shaker to excite the structure for easy experimentation. As compared to ambient vibration, use of a shaker has the advantage that the exact input force is known. Damage can be detected from both input and output data, making identification accuracy high. The problem with using a small shaker is that the excitation force is weak and measurements might be contaminated by ambient vibrations. To deal with this problem, we now developing a technique with which to extract the responses to harmonic excitation. The combination of the response extraction technique and proposed damage identification method will make possible the use a small shaker on a real structure.

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