Impact Location for a Multilayer Transversely Isotropic Plate

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ABSTRACT:

This paper addresses analytical research work performed on impact location in an anisotropic multilayered thermal protection structure. The propagation of elastodynamic waves through the multilayer structure and the possibility that the layers are transversely isotropic are two complications that are addressed. The method used is an extension of classical triangulation based on the use of the fixed geometry of the multilayer plate structure, quasi-longitudinal modes, and the generalized Snell's law for transversely isotropic materials. The impact localization problem is recast as a minimization problem whose objective function is the sum of the distances between all combination pairs of constant time difference curves for the sensor pairs. An initial estimate is made for the parameter values for the collection of constant time difference curves. The Newton-Kantorovich algorithm is then used to generate a sequence of iterations which converge to the minimum of the objective function. This generates a point on each curve in the collection. The centroid of this point set is calculated and used as an estimate of the impact point.

INTRODUCTION

Thermal protection systems on aerospace vehicles are exposed to possible damage by impact from debris, maintenance equipment, etc. The ability would exist in an ideal system to detect that an impact event has occurred, where the impact has occurred, quantify the impact energy, and determine the extent to which damage has occurred. A passive system to accomplish these goals using the acoustic energy generated by the impact itself would be desirable. The subject of the current work is the second level: impact localization.

A thermal protection system (TPS) is usually composed of an outer refractory layer with intermediate bonding or strain isolation layers which tie it to the vehicle outer surface.

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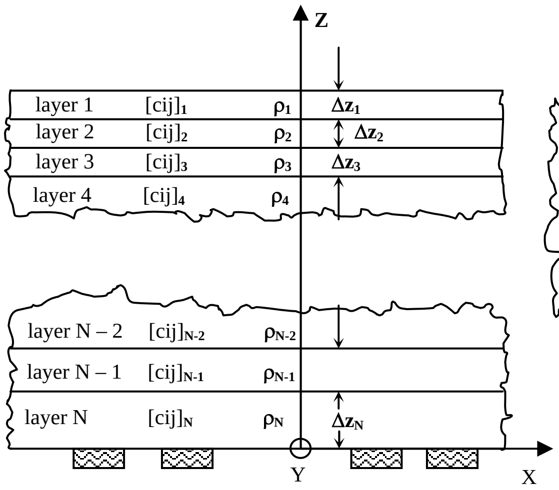

Figure 1. Multilayer Transversely Isotropic Plate(side).

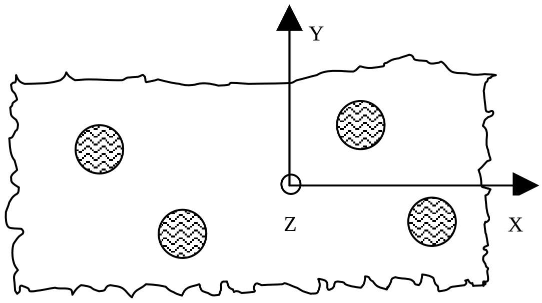

Figure 2. Multilayer Transversely Isotropic Plate(bottom).

Due to the extreme environment on the exterior of the TPS the use of backside sensing is considered to be practical. Thus the signals that will be detected from an impact will have to pass thru this layered structure. Some of the materials in this structure may also be anisotropic. The ceramic foam tile used in some TPS is transversely isotropic. This leads to the problem illustrated in Figure 1 and 2.

It is assumed that the impact generates signals that propagate out from the location of impact through the outermost layer media. The impact signals detected by the sensors have a propagation delay and the actual time an impact has occurred is unknown.This problem is similar in character to the classic location problem in an isotropic medium where two propagation observations are required if the time of signal initiation is known or three are required if the initiation time of the signal is unknown. In this case, the propagation time is directly proportional to the propagation distance and may be directly solved for. The layered structure and anisotropic material properties make the explicit analytic statement of the relationship between propagation time and distance difficult, if not impossible. This makes a numerical approach for finding a solution very attractive. This formulation may be viewed as a Model Based Inverse problem. If the location of impact and initial propagation direction is specified then the time to propagate through the layer assembly and exit point on the backside surface may be calculated. This would be the forward problem. The inverse problem consists of finding the coordinates of the impact location from knowledge of the relative arrival times at various monitoring points on the backside surface.

THEORY

A method of finding the forward solution must be generated first. The approach taken assumes that ray tracing is applicable and, as previously mentioned, that the impact signals move out in all half space directions. It is instructive to consider a series of

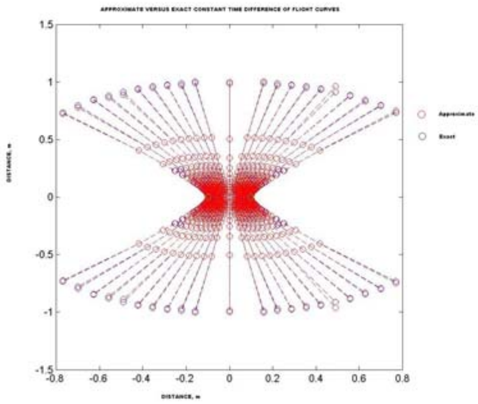

Figure 3 Time versus radial distance curve

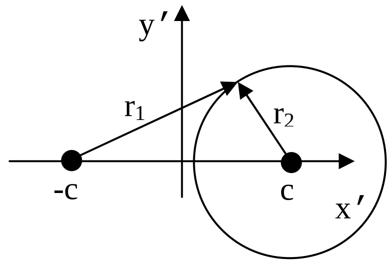

Figure 4 Local Geometry for Sensor Pair

problems of increasing difficulty to understand the complications added to the isotropic location problem. These are the thick isotropic plate problem, the multilayer thick isotropic plate problem, the thick anisotropic plate problem, and the multilayer thick anisotropic plate problem. The thick isotropic plate problem is directly solvable analytically. An alternate method, however, may be used. It is noticed that, once the thickness is fixed, the propagation time is a function of

$$r = \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2}$$

only, where (x_i, y_i) and (x_s, y_s) are the coordinates of the impact location and a sensor location, respectively. This information is used to generate a curve of the propagation time, *t*, versus the radial distance, *r*, from the impact axis to the sensing location. This *t(r)* curve, shown in figure 3, has a non-zero value of *t* for a zero radial propagation distance. Also, the slope of the curve is not constant even though the material is isotropic.

Once the *t(r)* function is found, the inverse function *r(t)* can be generated. If the transit times of the impact signal to two sensors were known then the backward problem could be solved by using *r(t)* and using triangulation. Additional information is required since the initial impact time is unknown. What may be found from the relative arrival times for the two sensors is the locus of possible impact points. The constant time difference curves are constructed in the sensor pair local coordinate system shown in figure 4 in the parametric form

$$x' = x'(s), \quad y' = s\,.$$

The curve for a particular time difference is created by assuming a propagation time to one of the sensors. The minimal value of this propagation time, $t(r_1)$, is determined by the distance, $2c$, between sensors. The point of possible impact on the line connecting the sensors is

$$|\Delta t| = t\left(2c - r_1^*\right) - t\left(r_1^*\right),$$

where r_1^* is the minimum value of r_1. Once r_1^* is found, other points on the curve may be found by gradually increasing r_1 and solving:

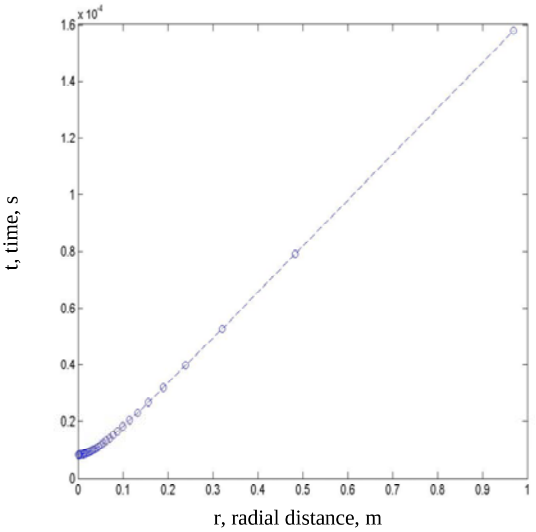

Figure 5. Family of constant time difference curves for a Sensor Pair.

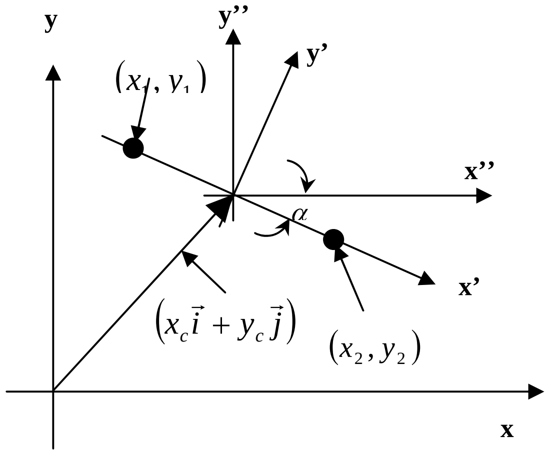

Figure 6. Relationship of Sensor Local Coordinates to Global Coordinates.

$$r_2 = r\left(\left|\Delta t\right| - t(r_1)\right)$$

When the curves are transformed into the global coordinate system (figure 5) the parametric equations become

$$\begin{bmatrix} x(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} \cos(a) & \sin(a) \\ -\sin(a) & \cos(a) \end{bmatrix} \begin{bmatrix} x'(s) \\ y'(s) \end{bmatrix} + \begin{bmatrix} x_c \\ y_c \end{bmatrix}.$$

By adding a third sensor there are three unique sensor pairs. Two of the sensor pairs may be used for the solution and the intersection of their possible impact curves is the impact point. If timing information is without error then the third sensor pair is redundant information because all of the curves will intersect at the same point. Otherwise, the third sensor may be used to improve the estimate of the impact point. If a point is selected on each curve as an estimate of the actual impact point, then one measure of the merit of the estimate would be the sum of the distances between the estimate points on these curves. This leads to an objective function

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\left(x_i(s_i) - x_j(s_j) \right)^2 + \left(y_i(s_i) - y_j(s_j) \right)^2 \right).$$

Minimizing this function will yield the point on each curve which is the best impact point estimate. A global best estimate can be found by taking the centroid of the set of curve estimate points. This approach allows additional sensors to be added for redundancy in a natural manner. For *N* sensors there will be *N(N-1)/2* unique sensor pairs. Using an explicitly differentiable representation for these curves allows the use of a gradient search method to minimize the objective function.

Adding one or more isotropic layers to the problem requires the calculation of an intercept point at the interface between the layers and the transmission angle for each consecutive layer. A point on the *t(r)* curve may be calculated for multiple isotropic layers by assuming an initial propagation angle. Using this propagation angle, the radial propagation distance and transit time are calculated for the current layer and the transmission angle into the next layer is calculated using Snell's Law. The transmission angle is used for calculating the radial propagation distance for the next layer, which is then used with Snell's law for calculating the transmission angle for the subsequent layer. This process is repeated until the final layer is processed. A running total of the radial distance and propagation time is kept. By using a range of initial angles, this 'shooting' method allows the construction of a series of points on the *t(r)* curve. If at any stage reflection occurs at the interface then the bottom of the assembly cannot be reached. The largest radial distance reached by the shooting method is considered to be the 'radius of observability'. The maximum 'shooting' angle used is capped at 80 deg. for practical reasons, even though 90 deg. is theoretically possible.

Consideration of the anisotropic layer requires the introduction of the concepts of the Christoffel equations, phase slowness, phase velocity, and group velocity. In an anisotropic material the propagation velocity of a mode is dependent upon the propagation direction. There are three independent propagation modes, as in an isotropic medium, but they are no longer pure modes. This means that, except for certain special directions, the modes are neither parallel nor perpendicular to the direction of maximal phase velocity. The direction of energy flux, the group velocity direction, is not parallel to phase velocity as with an isotropic medium. The phase slowness is the reciprocal of the phase velocity.

The Christoffel equations [1] allow the calculation of the phase slowness for a particular set of direction cosines. The Christoffel equations are:

$$\begin{vmatrix} \lambda_{11}-\rho c^2 & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22}-\rho c^2 & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33}-\rho c^2 \end{vmatrix} = 0$$

where:

$lij = \quad C_{ikji} n_k n_l$,
C_{iklj} is the stiffness tensor,
n_k is the k^{th} component of the direction cosine.

This eigenvalue problem yields three mode phase slowness magnitudes (eigenvalues) and directions (eigenvectors). Evaluation over all propagation directions gives a set of three closed surfaces as in Figure 7. Normally one surface is most closely aligned with the propagation direction, has the highest propagation velocity, and is known as the quasilongitudinal mode. There are two other modes known as quasishear. The phase slowness surfaces are spherical in an isotropic material. In a transversely isotropic material they are axisymmetric about the symmetry axis. Thus only the curves for any cross section passing through the symmetry axis need to be calculated.

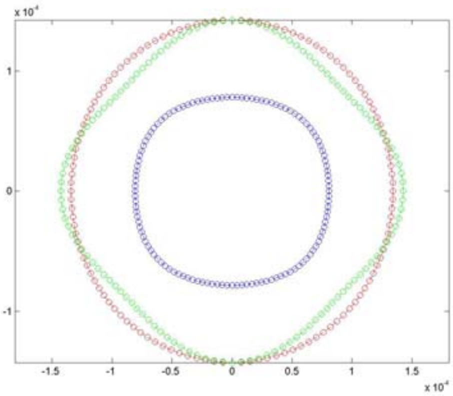

Figure 7. Phase Slowness Curves in XZ Plane for transversely isotropic material.

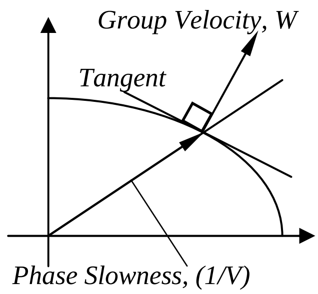

Figure 8. Relationship between Phase Slowness and Group Velocity Directions.

The group velocity direction [2], as shown in Figure 8, is normal to the phase slowness surface and its projection on the phase velocity direction vector has a magnitude equal to the phase velocity magnitude. The perceived propagation direction in an anisotropic medium is the group velocity. The phase velocity and group velocity surfaces may be constructed once the phase slowness surface is obtained. The *t(r)* curve for the layer may be found using the 'shooting method' with the group velocity.

Finally, the case of multiple anisotropic layers requires the use of the generalized Snell's law as shown in Figure 9. This results from the requirement that the transverse component of the phase slowness be equal at an interface between two dissimilar materials. The procedure for ray tracing through the assembly is to assume an initial propagation angle for the group velocity through the first layer. The radial propagation and transit time through the layer are then calculated. The point on the phase slowness surface corresponding to the group velocity direction is located. The generalized Snell's law is used to locate the corresponding point on the phase slowness surface of the next layer. The group velocity in the next layer is found from its association with the point on the phase slowness surface. This process is repeated until the bottom of the last layer is reached.

IMPLEMENTATION

This procedure has been implemented as an application in MATLAB®. The phase slowness curves of the materials in the model are generated by solving the Christoffel Equations in range of propagation directions in the XZ plane. The x and z components of the points on the curves are then parameterized in terms of ψ using nonlinear regression. This allows the tangent and normal vectors to be found from the derivatives of the parameterized coordinate equations.

Once the regression curves for the phase slowness components are generated the phase velocity curves can be calculated from the inverse of the phase slowness magnitude. The group velocity curves are then found by use of the normal to the phase slowness surface and the phase velocity magnitude. This information is stored in the form of group velocity magnitude and direction.

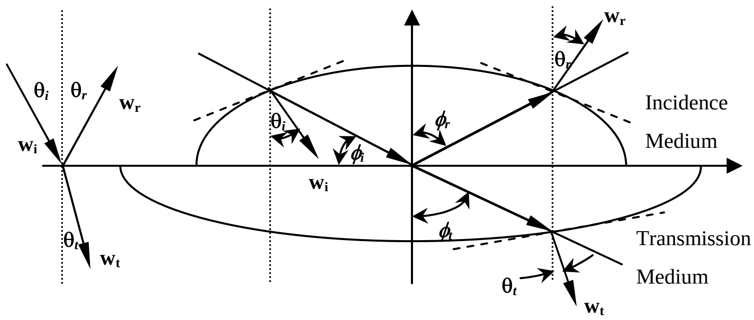

Figure 9. Generalized Snell's Law.

The group velocity information for each layer is used for determining the radial propagation distance and transit time. Linear interpolation is used when the propagation angle falls between known values. If there is more than one layer the group velocity and phase slowness information of touching layers is used together for solution of the generalized Snell's law relation. Once again linear interpolation is used.

Upon finishing the time versus radial distance curve, the constant time of flight difference curves are generated for each sensor pair. This is performed by observing which sensor has the smallest absolute time of arrival. This indicates which sensor is closest to the intercept of the constant time of flight curve and the sensor centerline. This intercept point is found using bisection. The other points on the constant time of flight curve within the radius of observability are then calculated by using linear interpolation to solve the relation between r_1 and r_2 given previously.

These curves are approximated over the radius of observability using Chebyshev polynomials[3]. The Chebyshev polynomials are then converted to regular polynomials using the method outlined in [4]. The results of the approximation are shown in figure 5. The regular polynomials are then converted from the local sensor pair coordinates into global coordinates using the transformation previously given.

These regular polynomial curves are used to construct the objective function. . The set of coordinate parameter values that minimize the objective function are found using the Newton-Kantorovich algorithm[5] as follows. An initial guess of the coordinate parameter for each curve is made. The Hessian matrix and gradient of the objective function are then evaluated at the current coordinate parameter values. The Hessian matrix inverse and the gradient vector are then used to create an updated coordinate parameter estimate. This is continued until convergence criteria are met or divergence is detected. If convergence occurs, then the final estimate of the impact location is found by evaluating the coordinates of each sensor pair curve from the current coordinate parameter values and then finding the centroid of this set of values as illustrated in Figure 11.

RESULTS

The method was evaluated using simulated timing data for layers of isotropic and anisotropic materials. It was also tested against experimental data gathered from impacts on an aluminum block. Figures 10 and 11 show graphical output from the program for a typical test data case.

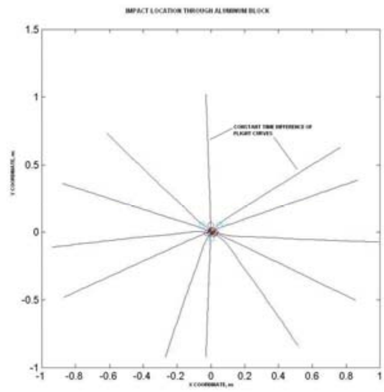

Figure 10. Impact Location Algorithm output for Observable Region.

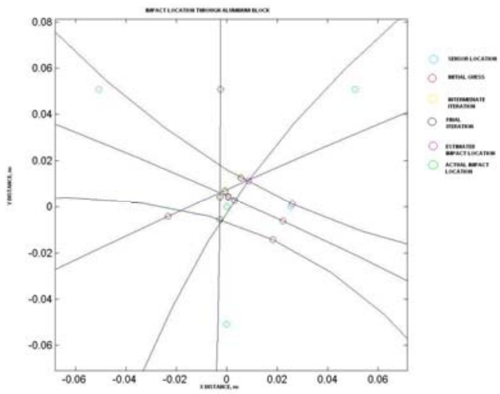

Figure 11. Close Up of Impact Location Region for Algorithm output.

These figures show the sensor locations, the actual impact point, the constant time difference curves generated for the sensor pairs from the timing data, the evaluation of the initial estimates on the curves, the intermediate estimates, the converged impact location estimates, and the final impact location estimate. Figure 10 shows these entities over the entire observable area. Figure 11 shows a close-up of the impact location area.

CONCLUSIONS

A method of finding impact location by using timing data from sensors residing on the backside of an assembly composed of multiple layers of transversely isotropic material has been formulated. This method works by solving a model based inverse problem where finding the propagation time through a transversely isotropic multilayer assembly is the forward problem. The method has been implemented in software and tested against simulated data for the multilayer transversely isotropic case and actual data for the single layer isotropic case.

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