

# Auxiliary Particle Filter for Structural Damage Identification

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## ABSTRACT:

The identification of structural damage is an important objective of health monitoring for civil infrastructures. Frequently, damage to a structure may be reflected by a change of some system parameters, such as a degradation of the stiffness. In this paper, an auxiliary particle filter (APF) method is proposed to identify a non-stationary dynamic system with abrupt change of system parameters. In the APF, the importance density is proposed as a mixture density that depends upon the past state and the most recent observations, thus which has a good time tracking ability. It is more suitable for tracking the non-stationary system than the conventional particle filters. Simulation results for tracking the parametric non-stationary changes of non-linear hysteretic structures are presented to demonstrate the application and effectiveness of the proposed technique in detecting the structural damages.

## INTRODUCTION

In the field of civil engineering, real-time structural identification of dynamic system subjected to earthquake motion has been focused on the accurate prediction as well as structural health monitoring and damage assessment. System identification and damage detection based on measured vibration data have received intensive studies recently. As an online identification method, the Kalman filter (KF) has received much attention and has successfully in the parameter estimation problems over the past years [1]-[4]. Similar methods such as least-square estimation method (LSE) [5]-[9], suboptimal  $H_\infty$  filter method [10], and unscented Kalman filter method [11] have been developed in some useful forms for solving many practical problems in civil engineering. To date, the online detection of the changes of structural parameters due to structural damages during a severe event, such as the earthquake, is still a challenging problem.

A filtering method, called particle filter (PF), also called bootstrap filter, based on Bayesian state estimation and Monte-Carlo method was proposed by Gordon [12], which

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has the great advantage of being able to handle any functional non-linearity and system and/or measurement noise of any distribution. Another similar method is the Monte-Carlo filter proposed by Kitagawa [13]. Because the particle filter offers a general numerical tool to approximate the state a posterior density in nonlinear and non-Gaussian filtering problems with arbitrary accuracy, it has quickly become a popular tool in signal processing applications[14]-[16]. This filter has been successfully used in radar tracking [17], structural parameters estimation [18] [19]. However, some of its problems, in particular those related to optimal sampling from the posterior distribution, which leads to choice of sampling importance distribution, efficiency of implementation and choice of an observation model, still remain. The most common choice of importance density is the transition prior density function for particle filter, since it is intuitive and simple to implement, but using the prior as the importance density suffers from drawback of without any knowledge of the observations, and hence the state space is explored without direct knowledge of the observations. Therefore, many particles are either wasted in low likelihood area, resulting in a low efficiency of the sampling, resulting in estimating failures. The particle filter may be not preferable for damage detection, because structural damage must be non-stationary phenomenon.

To accomplish this, it is necessary to incorporate the current observation in the importance density. A very elegant solution to this problem of optimally sampling from the posterior has been given by the so-called auxiliary particle filter (APF)[20]. The APF can be regarded as a one-look-ahead procedure. The main idea is to increase the influence of particles with a large predictive likelihood by choosing an importance density that takes the information from the current measurement into account.

In this paper, a Bayesian filtering method for structural damage identification based on the APF is developed. The APF has the great advantage is that it naturally generates the particles from the samples at the previous time step, which conditioned on the current measurement, become much dependent on the data observed nearest time, are most likely to be close to the true state. Such an adaptive tracking technique yields a sparse approximation and gives a larger importance to more recent data in order to cope with the system parameter's variations. The proposed technique is capable of tracking the non-stationary changes of system parameters from which the event and severity of structural damage may be detected online. Simulation results demonstrate that the proposed method is suitable for tracking the changes of system parameters for hysteretic structures.

## BAYESIAN FILTERING

The nonlinear non-Gaussian filtering problem we consider consists of computing the a posteriori density of the state vector, given the observed measurements. In a general discrete-time stochastic system model, the evolution of the state sequence  $\{\mathbf{x}_k, k \in N\}$  of the system given by

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}) \quad (1)$$

where  $\mathbf{f} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}^{n_x}$  is a possibly nonlinear function of the state  $\mathbf{x}_{k-1}$ ,  $\{\mathbf{v}_{k-1}, k \in N\}$  is an i.i.d. process noise sequence,  $n_x, n_v$  are dimensions of the state and process noise

vectors, respectively, and  $\mathbb{N}$  is the set of natural numbers. The objective of system is to recursively estimate from measurement

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{n}_k) \quad (2)$$

where  $\mathbf{h}: \mathbb{R}^{n_x} \times \mathbb{R}^{n_n} \rightarrow \mathbb{R}^{n_z}$  is a possibly nonlinear function,  $\{\mathbf{n}_k, k \in \mathbb{N}\}$  is an i.i.d. measurement noise sequence, and  $n_z, n_n$  are dimensions of the measurement and measurement noise vectors, respectively. In particular, we seek filtered estimates of  $\mathbf{x}_k$  based on the set of all available measurements  $\mathbf{z}_{1:k} = \{\mathbf{z}_i\}_{i=1}^k$  up to time  $k$ .

The Bayesian filtering is to recursively calculate some degree of belief in the state  $\mathbf{x}_k$  at time  $k$ , given the data  $\mathbf{z}_{1:k}$  up to time  $k$ . Thus, it is required to construct the *pdf*  $p(\mathbf{x}_k | \mathbf{z}_{1:k})$ . Our aim is to estimate recursively in time the *pdf*  $p(\mathbf{x}_k | \mathbf{z}_{1:k})$ , which are given by two stages: prediction and update.

Assumed that  $\mathbf{x}_k$  as system model (1) is a Markov process of initial distribution  $p(\mathbf{x}_0 | \mathbf{z}_0) = p(\mathbf{x}_0)$  and  $p(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{z}_{1:k}) = p(\mathbf{x}_k | \mathbf{x}_{k-1})$ . Supposed that the required *pdf*  $p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1})$  at time  $k-1$  is available, the prediction stage involves system model (1) to obtain the prior *pdf* of the state at time  $k$  via the Chapman-Kolmogorov equation

$$p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k | \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1}) d\mathbf{x}_{k-1} \quad (3)$$

where the probabilistic model of the state evolution  $p(\mathbf{x}_k | \mathbf{x}_{k-1})$  is defined by the system model (1) and the known statistic of  $\mathbf{v}_{k-1}$ .

At time step  $k$ , a measurement  $\mathbf{z}_k$  that is conditionally independent given the state  $\mathbf{x}_k$  become available, and this may be used to update the prior density to obtain the required posterior density of the recurrent state via Bayes' rule

$$p(\mathbf{x}_k | \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1})}{p(\mathbf{z}_k | \mathbf{z}_{1:k-1})} \quad (4)$$

where

$$p(\mathbf{z}_k | \mathbf{z}_{1:k-1}) = \int p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) d\mathbf{x}_k \quad (5)$$

depends on the likelihood  $p(\mathbf{z}_k | \mathbf{x}_k)$  defined by the measurement model (2) and the known statistics of  $\mathbf{n}_k$ .

For linear Gaussian models, the integral of the recursion can be solved analytically with a finite dimensional representation leading to the Kalman filter recursion, where the mean and covariance matrix of the state are propagated. Generally, this recursive propagation of the posterior density is only a conceptual solution, and it cannot be determined analytically. Therefore, numerical approximations of the integral have been proposed. A recent important contribution is to apply simulation based methods from mathematical statistics, the sequential Monte Carlo methods, commonly referred to as particle filters.

## PARTICLE FILTERING METHODS

The particle filter is an attractive approach for implementing a recursive Bayesian filtering to the problem of computing intractable posterior densities by Monte Carlo (MC) simulations. The key idea is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on these samples and weights. As the number of samples becomes very large, this MC characterization

becomes an equivalent representation to the usual functional description of the posterior densities, and the particle filter approaches the optimal Bayesian estimate.

Let us introduce an arbitrary importance distribution  $\pi(\mathbf{x}_{0:k} | \mathbf{z}_{1:k}) > 0$  whenever  $p(\mathbf{x}_{0:k} | \mathbf{z}_{1:k}) > 0$ , from which it is easy to get samples called importance sampling. Given  $N$  i.i.d. random particles  $\{\{\mathbf{x}_{0:k}^{(i)}\}_{i=1}^N\}$  distributed according to  $\pi(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})$ , an approximation Monte Carlo estimate of the posterior density at  $k$   $p(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})$  is given by

$$p(\mathbf{x}_{0:k} | \mathbf{z}_{1:k}) \approx \sum_{i=1}^N \tilde{w}_k^{(i)} \delta(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^{(i)}) \quad (6)$$

where the normalized importance weights are defined

$$\tilde{w}_k^{(i)} = \frac{w_k^{(i)}(\mathbf{x}_k^{(i)})}{\sum_{j=1}^N w_k^{(j)}(\mathbf{x}_k^{(j)})} \quad (7)$$

where the importance weights are defined by

$$w_k^{(i)} = \frac{p(\mathbf{x}_{0:k}^{(i)} | \mathbf{z}_{1:k})}{\pi(\mathbf{x}_{0:k}^{(i)} | \mathbf{z}_{1:k})} \quad (8)$$

The choice of importance distribution (proposal function) is one of the most critical design issues in importance sampling algorithms. The preference for proposal functions that minimize the variance of the importance weights is advocated by Doucet *et al.* (2001) [21]. When the proposal distribution  $\pi_{opt}(\mathbf{x}_{0:k} | \mathbf{z}_{1:k}) = p(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{z}_k)$  minimizes the conditional variance of the importance weights ( $\text{var}_{\pi_{opt}}[w_k^{(i)} | \mathbf{x}_{k-1}^{(i)}, \mathbf{z}_k] = 0$ ). Hence, with the assumptions of the states correspond to a Markov process and the observations are conditionally independent given the states, the weight update equation leads to

$$w_k^{(i)} = w_{k-1}^{(i)} p(\mathbf{z}_k | \mathbf{x}_{k-1}^{(i)}) \quad (9)$$

However, this proposal distribution suffers from certain drawbacks: first, it requires sampling from  $p(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{z}_k)$ , that may be difficulty, the other is calculation of the importance weights as specified in Eq.(9) that require evaluating the integral  $p(\mathbf{z}_k | \mathbf{x}_{k-1}^{(i)}) = \int p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{x}_{k-1}^{(i)}) d\mathbf{x}_k$  that may be analytically intractable.

It should be pointed out that there is no universal choice for proposal distribution, which is usually problem dependent. A popular choice among practitioners is so-called prior transition distribution  $\pi(\mathbf{x}_k | \mathbf{z}_{1:k}) = p(\mathbf{x}_k | \mathbf{x}_{k-1})$  for its easy implement, although it may be far from optimal, this choice of proposal distribution has been advocated by many researchers[12][13][22]-[25]. For this particular choice of importance distribution, it evident that the weights are given by

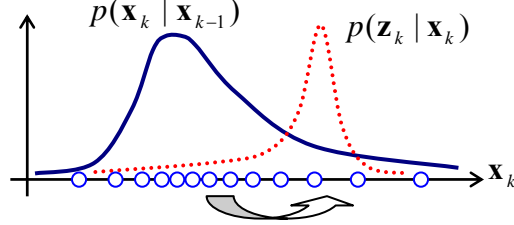
$$w_k^{(i)} \propto w_{k-1}^{(i)} p(\mathbf{z}_k | \mathbf{x}_{k-1}^{(i)}) \quad (10)$$

The transition prior sampling method does have the advantage that the importance weights are easily evaluated and easy to sample from. However, using the transition prior as the importance sampling density is independent of measurement, the state space is explored without any knowledge of the observations  $\mathbf{z}_k$ . Therefore, this filter may be inefficient and is sensitive to outliers and lead to poor performance. It results in higher Monte Carlo variation than the optimal proposal importance distribution.

Fig. 1 demonstrates why using  $p(\mathbf{x}_k | \mathbf{x}_{k-1})$  as the proposal distribution lead to poor performance. Sampling only from the prior can lead to imprecise estimates because the variability of the weights values  $w_k^{(i)}$  increases rapidly with time. In the figure, the likelihood  $p(\mathbf{z}_k | \mathbf{x}_k)$  is much more peaked than  $p(\mathbf{x}_k | \mathbf{x}_{k-1})$  or has little overlap with the prior. Thus, if we were to use the prior as the proposal distribution for this case, Eq.(10) indicates that many of the resulting samples  $\mathbf{x}_k^{(i)}$  could have negligible weights. In cases

where the variance of the system noise is significantly greater than the variance of the measurement noise, the prior tends to be poor choice for the proposal distribution.

As illustrated in the Fig.1, if we fail to use the latest available information to propose new values from the states, only a few particles might survive. It is therefore of paramount importance to move the particles towards to the regions of high likelihood. To achieve this, the proposed importance density should be included the information of the observations.



**Fig.1:** The APF proposal density allows us to move the particles in the prior to regions of high likelihood.

## AUXILIARY PARTICLE FILTERING

The APF was originally introduced by Pitt and Shephard [20] that operates by obtaining a sample from the joint density  $\pi(\mathbf{x}_k, i | \mathbf{z}_{1:k})$ , where  $i$  is the auxiliary variable that represents the index of the particle at  $k-1$  from which  $\mathbf{x}_k$  is predicted. The APF can be understood as a one-step ahead filtering: the particle  $\mathbf{x}_{k-1}^{(i)}$  is propagated to  $i^{(j)}$  in the next time step in order to assist the sampling from the posterior.

Using Bayes' rule,  $p(\mathbf{x}_k, i | \mathbf{z}_{1:k})$  can be expressed as

$$\begin{aligned} p(\mathbf{x}_k, i | \mathbf{z}_{1:k}) &\propto p(\mathbf{z}_k | \mathbf{x}_k, i) p(\mathbf{x}_k, i | \mathbf{z}_{1:k-1}) \\ &= p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | i, \mathbf{z}_{1:k-1}) p(i | \mathbf{z}_{1:k-1}) \end{aligned} \quad (11)$$

The APF operates by obtaining a sample from the joint density  $p(\mathbf{x}_k, i | \mathbf{z}_{1:k})$  and then omitting the indices  $i$  in the pair  $(\mathbf{x}_k, i)$  to produce a sample  $\{\mathbf{x}_k^{(j)}\}_{j=1}^N$  from the marginalized density  $p(\mathbf{x}_k | \mathbf{z}_{1:k})$ . Corresponding to (Eq.11), the importance density is chosen as a factorized form

$$\pi(\mathbf{x}_k, i | \mathbf{z}_{1:k}) = \pi(i | \mathbf{z}_{1:k}) \pi(\mathbf{x}_k | i, \mathbf{z}_{1:k}) \quad (12)$$

where

$$\pi(i | \mathbf{z}_{1:k}) \propto p(\mathbf{z}_k | \boldsymbol{\mu}_k^{(i)}) w_{k-1}^{(i)} \quad (13)$$

$$\pi(\mathbf{x}_k | i, \mathbf{z}_{1:k}) = p(\mathbf{x}_k | \mathbf{x}_{k-1}^{(i)}) \quad (14)$$

where  $\boldsymbol{\mu}_k^{(i)}$  is some characterization of  $\mathbf{x}_k$  given  $\mathbf{x}_{k-1}^{(i)}$ , this could be the conditional mean,  $\boldsymbol{\mu}_k^{(i)} = \mathbb{E}[\mathbf{x}_k | \mathbf{x}_{k-1}^{(i)}]$  or a sample  $\boldsymbol{\mu}_k^{(i)} \sim p(\mathbf{x}_k | \mathbf{x}_{k-1}^{(i)})$ .

Thus we can sample from  $\pi(\mathbf{x}_k, i | \mathbf{z}_{1:k})$  by resampling with replacement from the sample set  $\{\mathbf{x}_k^{(j)}, i^{(j)}\}_{j=1}^N$  that has an importance weight proportional to

$$w_k^{(j)} \propto \frac{p(\mathbf{x}_k^{(j)}, i^{(j)} | \mathbf{z}_{1:k})}{\pi(\boldsymbol{\mu}_k^{(i^{(j)})}, i^{(j)} | \mathbf{z}_{1:k})} \propto \frac{p(\mathbf{z}_k | \mathbf{x}_k^{(j)})}{p(\mathbf{z}_k | \boldsymbol{\mu}_k^{(i^{(j)})})} \quad (15)$$

## EXAMPLES

Consider an  $m$  degree of freedom (DOF) non-linear hysteretic shear-type structure subject to ground excitation  $\ddot{u}_g$ , the equation of motion is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}) = -\mathbf{M}\{\mathbf{I}\}\ddot{u}_g \quad (16)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  are the mass and damping matrices;  $\mathbf{x}$ ,  $\dot{\mathbf{x}}$ , and  $\ddot{\mathbf{x}}$  are the relative displacement, velocity, and acceleration vector to the ground;  $\{\mathbf{I}\}$  is the identity of the  $m \times 1$  column matrix; and  $\mathbf{f}$  the restoring force vector expressed by the Bouc-Wen model [26]. In this case, the  $i$ -th component of the vector is

$$\dot{f}_i = k_i \dot{u}_i - \alpha_i |\dot{u}_i| |f_i|^{n_i-1} f_i - \beta_i \dot{u}_i |f_i|^{n_i} \quad i=1, \dots, m \quad (17)$$

where  $\dot{u}_i = \dot{x}_i - \dot{x}_{i-1}$  is the relative velocity between the  $i-1$ -th and  $i$ -th mass point; and  $c_i, k_i, \alpha_i, \beta_i$  and  $n_i$  are the damping, stiffness and the non-linear parameters of the  $i$ -th mass point.

Regarding the unknown parameters as state variables, one can define an augmenting state vector  $\mathbf{X}$  as

$$\mathbf{X} = \{\dots, \dot{u}_i, f_i, c_i, k_i, \alpha_i, \beta_i, \log_{10}^{n_i}, \dots\}^T, \quad i=1, \dots, m \quad (18)$$

in this state, to ensure positivity of the parameter  $n_i$ ,  $\log_{10}^{n_i}$  rather than  $n_i$  is included in the augmented state vector. Eqs.(16) and (17) can then be rewritten in the form of non-linear state equations

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}) + \mathbf{v} \quad (19)$$

$$\text{where } \mathbf{F}(\mathbf{X}) = \left\{ \begin{array}{c} \vdots \\ -\frac{c_i}{m_i} \dot{u}_i - \frac{f_i}{m_i} - \frac{(1-\delta_{im})(c_{i+1} \dot{u}_{i+1} + f_{i+1})}{m_i} - \ddot{u}_g \\ k_i \dot{u}_i - \alpha_i |\dot{u}_i| |f_i|^{n_i-1} f_i - \beta_i \dot{u}_i |f_i|^{n_i} \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{array} \right\}, \quad \delta_{im} = 0 (i \neq m), \quad \delta_{im} = 1 (i = m) \quad \text{and } \mathbf{v}$$

is the process noise vector.

The observation equation here is expressed as

$$\mathbf{Z} = \mathbf{H}\mathbf{X} + \mathbf{n} \quad (20)$$

where  $\mathbf{n}$  is the observation noise vector, in which  $\mathbf{Y}$  is the observation defined by

$$\mathbf{Z} = \{\dots, \dot{u}_i, \dots\}^T, \quad i=1, \dots, m \quad (21)$$

and  $\mathbf{H}$  measurement matrix given by

$$\mathbf{H} = \begin{bmatrix} \ddots & & 0 \\ & \mathbf{h}_i & \\ 0 & & \ddots \end{bmatrix}, \quad i=1, \dots, m \quad (22)$$

where

$$\mathbf{h}_i = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \quad (23)$$

Utilising the auxiliary particle filtering technique in Eqs. (19) and (20), the state vector  $\mathbf{X}_k$  can be estimated from the input  $\ddot{u}_g$  and the observed output  $\mathbf{Z}_k$ . Hence, the unknown parameters are estimated simultaneously.

TABLE 2. Initial conditions ( $\mathbf{X} \sim \mathcal{N}(\mathbf{X}_0, \boldsymbol{\sigma}^2)$ )

$\mathbf{X}$	$\dot{u}_1$	$f_1$	$c_1$	$k_1$	$\alpha_1$	$\beta_1$	$\log_{10}^n$	$\dot{u}_2$	$f_2$	$c_2$	$k_2$	$\alpha_2$	$\beta_2$	$\log_{10}^{n_2}$
$\mathbf{X}_0$	0	0	1.05	36.75	0.86	0.65	0.2	0	0	0.85	29.4	0.55	0.65	0.2
$\boldsymbol{\sigma}^2$	$0.01^2$	$0.01^2$	$0.07^2$	$2.45^2$	$0.15^2$	$0.15^2$	$0.02^2$	$0.01^2$	$0.01^2$	$0.07^2$	$2.45^2$	$0.15^2$	$0.15^2$	$0.02^2$

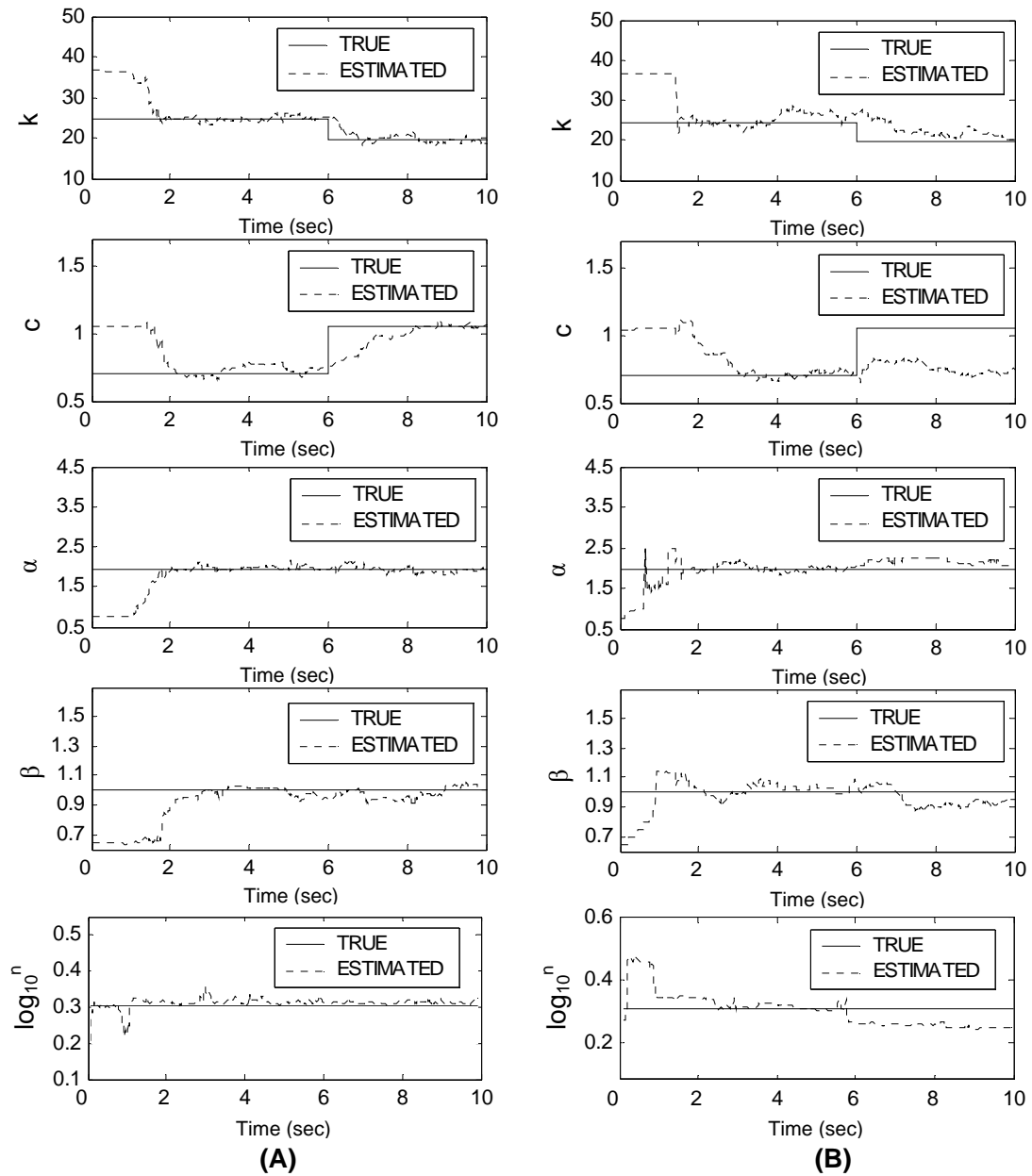
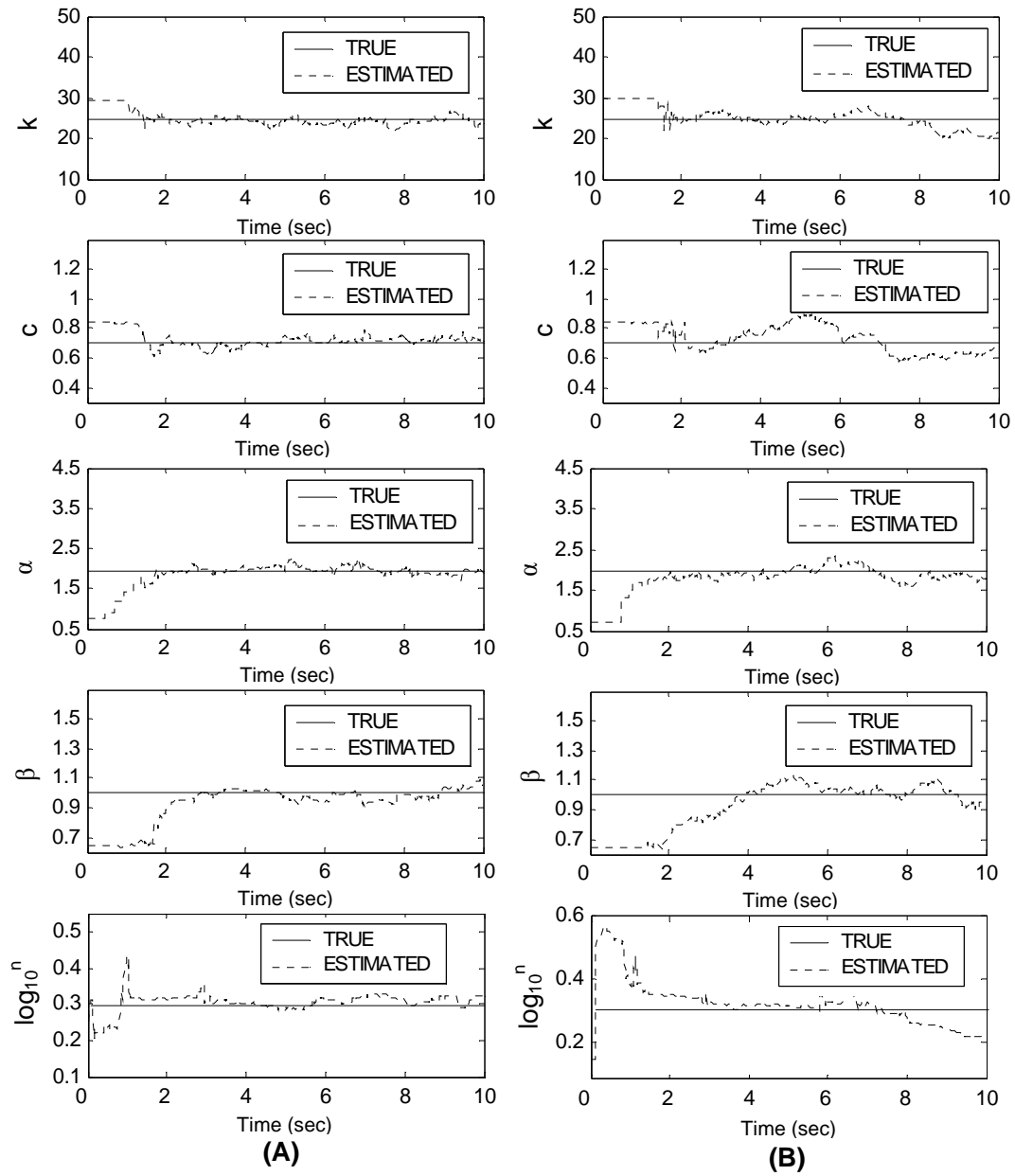


Fig. 2. Identified parameters  $k$ ,  $c$ ,  $\alpha$ ,  $\beta$  and  $\log_{10}^n$  for a 2-DOF hysteretic structure with abruptly changed parameters (1st story): (A) auxiliary particle filter method, (B) particle filter method.



**Fig. 2.** Identified parameters  $k$ ,  $c$ ,  $\alpha$ ,  $\beta$  and  $\log_{10}^n$  for a 2-DOF hysteretic structure with abruptly changed parameters (2nd story): **(A)** auxiliary particle filter method, **(B)** particle filter method.

To explore the effectiveness of the APF algorithm, we consider a 2-story shear-beam building ( $m=2$ ) subject to earthquake excitation. The El Centro (NS, 1940) earthquake record with the modified maximum amplitude of  $25 \text{ cm/sec}^2$  is the input excitation. The structural responses sampling interval is 0.01s. In this building, two story units are the Bouc-Wen model is used. The properties of each story unit are:  $m_1 = m_2 = 125.53 \text{ kg}$ ,  $c_1 = c_2 = 0.7 \text{ kN s/m}$ ,  $k_1 = k_2 = 24.5 \text{ kN/m}$ ,  $\alpha_1 = \alpha_2 = 2$ ,  $\beta_1 = \beta_2 = 1$ ,  $n_1 = n_2 = 2$ . Suppose a damage just occurs in the 1st story unit at  $t = 6 \text{ s}$ , at which time the stiffness in the first story unit  $k_1$  reduces abruptly from 24.5 to 19.6 kN/m, and the damping  $c_1$  increases abruptly from 0.7 to 1.05 kNs/m. Table 1 shows the initial conditions of the structural parameters for this



analysis. The process and observation noises are defined by  $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q})$  and  $\mathbf{n}_k \sim \mathcal{N}(0, \mathbf{R})$ , where  $\mathbf{Q} = \text{diag}(0.001^2, 0.001^2, 0.007^2, 0.3^2, 0.02^2, 0.01^2, 0.001^2, 0.001^2, 0.007^2, 0.3^2, 0.02^2, 0.01^2)$  and  $\mathbf{R} = \text{diag}(0.0025, 0.0025)$ .

Based on the proposed tracking technique, the identified results with the total number of particle realizations  $N=600$  are presented in Fig. 2 (A) and Fig. 3 (A). Also shown in Fig. 2 (B) Fig. 3 (B) are estimation results by particle filter for comparison. It is observed from Fig. 2 and Fig.3 that the proposed method tracks the structural parameters and their variations very well. Also shown in these figures, the APF has a good time tracking ability is more suitable for tracking the non-stationary system than the conventional particle filters.

## CONCLUSIONS

The auxiliary particle filter algorithm offers the ability to incorporate the current measurement into the proposal distribution, which essentially performs resampling before state prediction and weight update, as opposed to the traditional particle filter that uses the transition prior as proposal distribution, which samples from the prior without any knowledge of the current measurements. The auxiliary particle filter technique has been proposed to identify online the structural parameters and their variations due to damages for non-linear hysteretic structures. It is shown that the proposed method consistently achieves a better level of accuracy for estimating and tracking the parameters and their abrupt changes than the traditional particle filter method. Numerical results indicate that the proposed approach is particularly suitable for tracking the abrupt changes of system parameters from which the structural damage can be determined.

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