Study of Structural Damage Identification Based on Information Fusion and Shannon Entropy

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ABSTRACT:

Vibration-based damage identification is a useful tool for structural health monitoring. However, the uncertainties caused by measurement noise, modeling error involved in an analytical model, and environmental changes such as variations in temperature and load conditions can impede reliability of damage identification. In this paper, information fusion based on D-S (Dempster-Shafer) evidence theory and Shannon entropy are employed for decreasing the uncertainty and improving accuracy of damage identification. Regarding that the multiple evidence from different information sources are different importance and not all the evidences are effective for the final decision. The different importance of the evidences is considered by assigning weighting coefficient. Shannon entropy is a measurement of uncertainty. In this paper it is employed to measure the uncertainty of damage identification results. The first step of the procedure is training several artificial neural networks with different input parameters to obtain the damage decisions respectively. Second, weighing coefficients are assigned to neural networks according to the reliability of the neural networks. The Genetic Algorithm is employed to optimize the weighing coefficients. Third, the weighted decisions are assigned to information fusion center. And in fusion center, a selective fusion method is proposed. Numerical studies on the Binzhou Yellow River Highway Bridge are carried out. The results indicate that the method proposed can improve the damage identification accuracy and increase the reliability of damage identification to compare with the method by neural networks alone.

INTRODUCTION

During the past two decades, many research works have been conducted in the area of damage detection based on dynamic characteristics with different algorithms (Doebling et al. 1996). Vibration-based damage identification is a useful tool for

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structural health monitoring. But, the uncertainties caused by measurement noise, modeling error involved in an analytical model, and environmental changes such as variations in temperature and load conditions can impede reliable identification of damage (Farrar, 1997). Therefore, new techniques to solve this problem should be developed.

In recent years information fusion technique attracts increasing attentions to structural health monitoring due to its inherent capabilities in extracting information from different sources and integrating them into a consistent, accurate and intelligible data set (Hall 1992, Kozinea et al. 2000). Some researchers are engaging in study of the damage identification methods using information fusion technique to achieve improved accuracies and more specific inferences. Guo (2005) use the information techniques to detect the damage of two-dimensional truss structure and study the effect of three main fusion approaches. Jiang (2005) discusses the feasibility of applying data fusion to the structural health monitoring and describes the complex structural damage detection techniques based on probabilistic neural networks and data fusion.

In this paper, information fusion based on D-S (Dempster-Shafer) evidence theory and Shannon entropy are employed for improving accuracy of damage identification.

DEMPESTER-SHAFER EVIDENCE THEORY

D–S evidence theory is a mathematical theory of evidence. The work on the subject is Shafer in 1976, which is an expansion of Dempster. In a finite discrete space, D-S evidence theory can be interpreted as a generalization of probability theory where probabilities are assigned to sets as opposed to mutually exclusive singles. In traditional probability theory, evidence is associated with only one possible event. In D–S evidence theory, evidence can be associated with multiple possible events.

Assume that Ω denotes the space of hypotheses. For a finite set of mutually exclusive and exhaustive propositions Ω, a power set 2^{Ω} is the set of all the subsets of Ω including itself and a null set, *Φ*. The basic probability assignment (BPA) is an important concept of evidence theory. Generally speaking, the term “basic probability assignment” does not refer to probability in the classical sense. For any hypothesis *A* of 2^{Ω}, the BPA is a function m: $2^{\Omega} \rightarrow [0, 1]$ such that:

$$m(\phi) = 0 \tag{1}$$

$$\sum_{A \in \Omega} m(A) = 1 \tag{2}$$

From the basic probability assignment, the upper and lower bounds of an interval can be defined. This interval contains the precise probability of a set of interest (in the classical sense) and is bounded by two non-additive continuous measures called belief function and plausibility function as shown in equation (3) and equation (4).

$$Bel(A) = \sum_{B|B \subseteq A} m(B) \tag{3}$$

$$Pl(A) = \sum_{B|B \cap A \neq \phi} m(B) \quad (4)$$

The measures of evidence (i.e. BPA) can be combined by Dempster's rule. Here multi information sources S_1, S_2,…, S_n are considered. Let $m_1(S_1)$, $m_2(S_2)$,…, $m_n(S_n)$ be basic probability assignment given by sources S_1, S_2, … S_n respectively. The combination rule is written as:

$$m(C) = (1-k)^{-1} \sum_{\cap S_i = C} \prod_{i=1}^{n} m_i(S_i) \quad (5)$$

$$k = \sum_{\cap S_i \neq \phi} \prod_{i=1}^{n} m_i(S_i) \quad (6)$$

where k represents basic probability mass associated with conflict. This is determined by the summing the products of the BPA of all sets where the intersection is null. (1-k) is used to compensate for the loss of non-zero probability assignments to non-intersecting subsets, and ensure that the probability assignments of resultant BPA also sum to 1.

SHANNON ENTROPY

Shannon defines entropy in terms of a discrete random event x, with possible states (or outcomes) n as:

$$H(x) = \sum_{i=1}^{n} p(i) \log_2 p\left(\frac{1}{p(i)}\right) = -\sum_{i=1}^{n} p(i) \log_2 p(i) \quad (7)$$

That is, the entropy of the event x is the sum, over all possible outcomes i of x, of the product of the probability of outcome i times the log of the inverse of the probability of i. This also can be applied to a general probability distribution, rather than a discrete-valued event.

Shannon shows that any definition of entropy satisfying the assumptions will be of the form:

$$H(x) = -K \sum_{i=1}^{n} p(i) \log_2 p(i) \quad (8)$$

Shannon's entropy measure came to be taken as a measure of the uncertainty about the realization of a random variable. Entropy has some special character as follows: In experiment A, if $p(\underline{i})$=1 and the rest equal zero, then H(x)=0. For there is no any uncertainty, a decisive conclusion can be made. On the contrary, if we know nothing about experimental results in advance, then $p(i)=1/n$, i=1, 2, 3,…, n, the maximum value of $H(x)$ was got:

$$H(x)_{\max} = K \log 2(n) \quad (9)$$

Obviously, in this case the result has the maximal uncertainty.

DAMAGE IDENTIFICATION PROCEDURES

The first step of procedures is training several artificial neural networks with different input parameters to obtain the damage decisions respectively. Second, weighing coefficients are assigned to trained neural networks according to the reliability of the neural networks. The Genetic Algorithm is employed to optimize the weighing coefficients. Third, the weighted decisions are sent to information fusion center. In fusion center, a selective fusion method was presented. Initially the high weight coefficient decision is chosen as the original decision. Then it is combined with the decision which has a second high weight coefficients by D-S theory for getting the first fusion decision. Next, the Shannon entropy of original decision and first fusion decision are analyzed. And the decision which has less entropy is selected to combine with the third decision for getting the second fusion decision. According to this principle, the last result was get. This procedure of damage identification above was called as weighted and selective fusion method.

Fitness function (object function) of Genetic Algorithms is set as total accuracy of the identification results. Some samples are needed to find the best or near best weight coefficients. The role of those samples is the same as the role of the training samples in a neural network. The function is presented in equation (10). The best identification results can be obtained through searching the optimal or approximate-optimal η_i (i =1, 2,…, n).

$$y = f(\eta_1, \eta_2, \ldots, \eta_n) \tag{10}$$

where y = the total accuracy of the identification results; η_i = weight coefficients (i= 1,2,..., n).

Suppose that $\Omega = \{A_1, A_2, \ldots, A_m, U\}$ is an effective frame of discernment. U is the uncertainty. Normalize the outputs of the neural networks and let them satisfies the request of the basic probability assignment function. The BPA of Ω can be get. That are $m_l(A_i)$, $m_l(U)$ =0 (i = 1, 2,…, m; l = 1, 2,…, n). n is the number of the neural networks to discern the Ω.

Different neural networks have different weight coefficients. So the BPA are weighted and adjusted like equation (11) and (12).

$$\overline{m}_l(A_i) = m_l(A_i) \times \eta_j \tag{11}$$

$$\overline{m}_l(U) = 1 - \sum_{i}^{m} \overline{m}_l(A_i) \tag{12}$$

where η_l = weighting coefficients. And $\eta_l \in [0, 1]$, l =1, 2,…, n.

The BPA weighted and adjusted still satisfies the request of BPA. Then they can be combined by Dempster's rule of combination. The combination rule is written as:

$$m(A_i) = (1-k)^{-1} \sum_{\cap A_i = A_i} \prod_{l=1}^{n} \overline{m}_l(A_i) = (1-k)^{-1} \sum_{\cap A_i = A_i} \prod_{l=1}^{n} \eta_l m_l(A_i) \tag{13}$$

$$k = \sum_{\cap A_i = \phi} \prod_{l=1}^{n} \eta_l m_l(A_i), \qquad i=1, 2, \ldots, m \tag{14}$$

The equation (14) shows that we can reduce the conflict of evidence from different information sources when $0<\eta_l<1$.

NUMERICAL EXAMPLES

A three-dimensional finite element model of the Binzhou Yellow River Highway Bridge as shown in Figure 1 is employed.

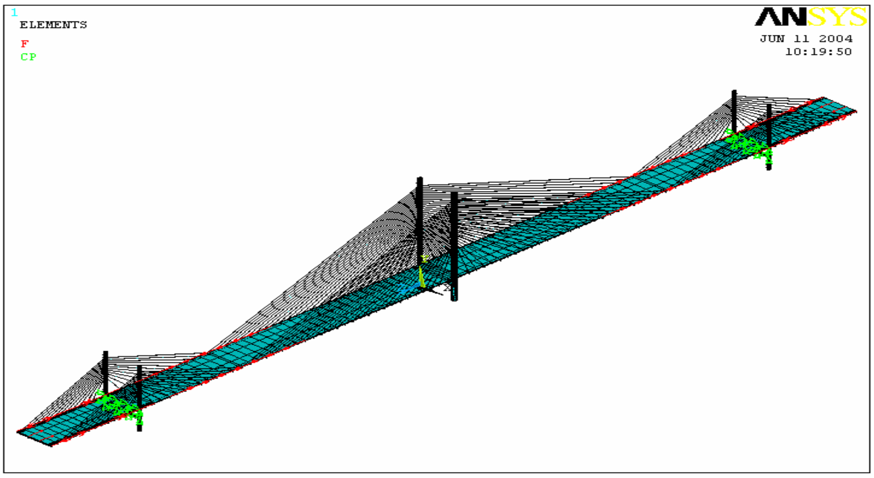

Figure 1: Finite element model of whole bridge

Generate the training samples for the artificial neural networks

The finite element model of the Binzhou Yellow River Highway Bridge is divided into 27 segments so as to easily to locate the damage. Twenty eight damage patterns, including twenty seven damage patterns matching one segment of bridge damaged and one pattern with no damage mated to 28 outputs of the artificial neural networks. The segments are shown in Figure 2.

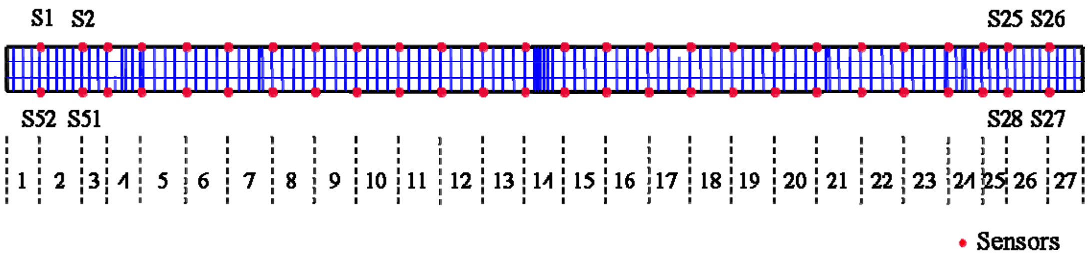

Figure 2: Location of test points

So as to get training samples, 1 damage serials are produced in one segment of the bridge finite element model for each damage pattern. The damages are simulated by reducing 40% of stiffness of longitudinal girders and deck elements. The first 6 frequencies, 4 vertical mode shapes and 2 torsional mode shapes are obtained. To simulate measurement uncertainties in the estimated modal parameters, the exact modal parameters, obtained from the analytical modal with the assumed damage, are perturbed with noise. More explicitly the estimated modal parameter set $\hat{\psi}(n)$ is constructed as:

$$\hat{\psi}(n) = \psi(1 + \frac{\varepsilon}{100} r) \tag{15}$$

where ψ = the exact modal parameter set obtained from the analytical model; r = normally distributed random number with zero mean and a variance of 1.0; ε = noise level in terms of percentage. 50 training samples were produced with noise level of 1% for every damage serials. There are 1400 training samples in all.

Train the artificial neural networks

Train the first artificial neural network (ANN1). The ratios of the first 7 mode shapes between before and after damages were used as the input to the ANN1.

$$\varphi_{i,j} = \frac{\left(\varphi_{i,j}^{u} - \varphi_{i,j}^{d}\right)}{\varphi_{i,j}^{u}} \times 100\% \tag{16}$$

where φ_{ij} = ratio of the mode shape; $\varphi^{u}_{i,j}$ = the value of jth mode shape at i test point before damage; $\varphi^{d}_{i,j}$ = the value of jth mode shape at i test point after damage.

Train the second artificial neural network (ANN2). Mode shape curvature (MSC) presented by Pandy et al is shown as follows:

$$\left\{\Delta\varphi''\right\}_j = \left|\left\{\varphi''_d\right\}_j - \left\{\varphi''_u\right\}_j\right| \tag{17}$$

First four vertical mode shapes and two torsional mode shapes are used to calculate the mode shape curvatures and used as the inputs of ANN2.

Train the third artificial neural network (ANN3). The flexibility matrix before and after damage can be expressed as:

$$[F]^u \approx \sum_{i=1}^{n} \frac{1}{\left(\omega_i^u\right)^2} \left\{\phi_i^u\right\}\left\{\phi_i^u\right\}^T \tag{18}$$

$$[F]^d \approx \sum_{i=1}^{n} \frac{1}{\left(\omega_i^d\right)^2} \left\{\phi_i^d\right\}\left\{\phi_i^d\right\}^T \tag{19}$$

where ω_i^u = the ith frequency before damage; ω_i^d = the ith frequency after damage; ϕ_i^u = the ith mode shape before damage; ϕ_i^d = the ith mode shape after damage. The variance of the stiffness matrix can be represented as:

$$\theta_j = \frac{F_{jj}^u - F_{jj}^d}{F_{jj}^u} \tag{20}$$

where θ_j = the variance of flexibility at j. Location corresponding the maximum value of θ_j is considered as damage place. The first 3 frequencies and mode shape are taken into account, so the ANN3 have 26 inputs.

The results

Just like the training samples, each damage pattern has one damage serials. Therefore there are 28 damage serials. In order to consider the effects of noise to the identification accuracy of the neural networks, different noise was added with seven levels: 2%, 4%, 6%, 8%, 10% and 15%. 50 test samples are produced randomly with each noise level. Each damage pattern has 1400 test samples, so there are 8400 test samples totally.

To find the optimal or approximate optimal weight coefficients, another group of test samples was produced under noise level of 3%, 7%. Total is 1400×2=2800 samples. The weight coefficients calculated by Genetic Algorithms are shown in Table 1. Each ANN test results and the fusion results are shown in Table 2.

Table 1 Weight coefficients

ANN	ANN1	ANN2	ANN3
Weight coefficients	9.8835000e-001	7.6020000e-001	6.6660000e-001

Table 2 Each ANN test results and the fusion result

Noise level	2%	4%	6%	8%	10%	15%	Total
ANN1	1400	1396	1297	1135	937	506	6671
Accuracy	100%	99.71%	92.64%	81.07%	66.93%	36.14%	79.42%
ANN2	1396	1303	1147	897	727	431	5901
Accuracy	99.71%	93.07%	81.93%	64.07%	51.93%	30.79%	70.25%
ANN3	1226	869	646	514	425	264	3944
Accuracy	87.57%	62.07%	46.14%	36.71%	30.36%	18.86%	46.95%
Direct fusion	1393	1217	985	839	707	510	5651
Accuracy	99.50%	86.93%	70.36%	59.93%	50.50%	36.43%	67.27%
Weighted and selective fusion	1400	1399	1377	1277	1112	693	7258
Accuracy	100%	99.93%	98.36%	91.21%	79.43%	49.50%	86.40%

It is shown in Table 2 that the traditional method of damage identification based on artificial neural network is sensitive to the ambient noise. The accuracy is not improved by directly combining the results of ANN1, ANN2 and ANN3 with D-S evidence theory. The total accuracy of direct fusion is 67.27%, reducing 12.15% compared with ANN1 which is best one in signal ANN. But the accuracy of weighted and selective fusion is 86.40%, increasing 6.98% compared with ANN1. From the above analysis, the method of weighted and selective fusion presented in this paper is more effective than both the direct fusion and the single ANN.

CONCLUSION

Taking into account the numerical investigations of the Binzhou Yellow River Highway Bridge, the conclusions can be drawn. First, the traditional method of damage identification based on artificial neural network is sensitive to the ambient noise. Second, it should be considered that the multiple evidence from different information sources are different important and not all the evidences are effective for the last decision. The Shannon entropy employed to measurement of uncertainty in this paper is appropriate. And the selective fusion strategy is useful to reduce the uncertainty of damage identification results. Third, the results indicate that the method proposed can improve the damage identification accuracy and increase the reliability of damage identification compare with the method by neural networks alone.

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