Damage Diagnosis of Fame Structure Using Modified Modal Strain Energy Change Method

TING-YU HSU[1] & CHIN-HSIUNG LOH[2]

ABSTRACT

A modified modal strain energy change (M-MSEC) method and its corresponding iteration process are presented to detect damage of frame structures. It improves that the damage quantification obtained by using different kind of modes in M-MSEC can be identified correctly. The effectiveness of the proposed algorithm is demonstrated via numerical study of a 3-D frame structure. A full scale experimental study is also performed to evaluate the robustness of the M-MSEC method on damage detection. Satisfactory results are shown in relating to the modeling error, noise effect and limited measurements.

INTRODUCTION

Structure damage will induce the change of modal information, such as natural frequencies, mode shapes, and modal damping. These modal characteristics have been utilized to detect the damage location and quantity, and several techniques have been proposed in resent years [1]. Modal strain energy (MSE), which is a function of mode shape and elemental stiffness, has been utilized initially as the indictor for modal selection [2, 3], and later has been treated as a damage indicator [4, 5]. Furthermore, the sensitivity of the modal strain energy change (MSEC) with respect to the local damage is derived, and is utilized to detect the location and quantity of damage [6]. The MSEC method was later improved, and the modal truncation error and the finite-element modeling error in higher modes were reduced [7]. The corresponding modal expansion for incomplete measured mode shapes, damage localization by modal strain energy change ratio (MSECR) and the threshold of elemental MSE has been discussed to increase the accuacy of MSEC method [8].

In practice, there are no elements with equally reduction of stiffness in each DOF unless the element is totally removed or damaged. For a beam-column element, the stiffness directly relates to the sectional properties. Therefore, the elemental stiffness matrix is consider as the combination of stiffness matrices contributed by different sectional properties, hence the original MSEC method can be modified to identify the

---

[1] Ph.D Student, Dept. of Civil Engineering, National Taiwan Univ., Taipei, Taiwan. E-mail: dysheu@ncree.org

[2] Professor, Dept. of Civil Engineering, National Taiwan Univ., Taipei, Taiwan. E-mail: lohc0220@ccms.ntu.edu.tw

sectional properties of elements. Therefore, the damage extent of elements can be identified more clearly by MSEC mtehod, and all kind of modes and their combination become a useful tool to identify the element damage quantity. Because the natural frequencies are determined much more accurately than the mode shapes, the sensitivity function of natural frequencies can also be added to the MSEC method.

When enough damage has occurred to cause the change of the modal parameters and stiffness matrix of the system, the relation between the MSEC and the damage reduction factor becomes nonlinear. Certain iteration process is required to obtain more accurate assessment of the severity of the damage. In this study a modified iteration process is proposed which considering the updated MSE target which is obtained from expanding the incomplete mode shapes in accordance with the current structure state. The traditional MSEC method did not consider this update mode shapes to evaluate the MSEC.

Besides, all the simulation and experimental studies for MSEC in previous papers solely focus on 2-D structures. Therefore, another main purpose of this study is to apply the MSEC to a 3-D structure with very limited measurements, which is common encountered in civil engineering structure. The numerical study using the finite element modal (FEM) of the 3-D test structure is performed first to verify the proposed modified MSEC method and the iteration process. The experimental study of this application to a 3-D test structure is also conducted to see the effectiveness of damage detection of the proposed method.

DAMAGE LOCALIZATION

Modal Strain Energy Change Ratio

The basic idea of MSE is defined as the product of the elemental stiffness matrix and the second power of the mode shape component. For the j th element in the i th mode, the MSE before and after the occurrence of damage is given as

$$MSE_{ij} = \Phi_i^T K_j \Phi_i \qquad \text{and} \qquad MSE_{ij}^d = \Phi_i^{dT} K_j \Phi_i^d \qquad (1)$$

where Φ_i is the i th mode shape of the undamaged state, K_j is the undamaged elemental stiffness of j th element, superscript T denotes the transpose, and the superscript d denotes the damaged state. In MSE_{ij}^d, K_j^d is replaced by K_j as an approximation since the damaged stiffness matrix is unknown in advance. The modal strain energy change ratio (MSECR) is defined as

$$MSECR_{ij} = \frac{MSE_{ij}^d - MSE_{ij}}{MSE_{ij}} \qquad (2)$$

without taking the absolute value in the numerator, and this non-absolute MSECR has been proven more suitable for damage localization than the absolute MSECR [8]. If a total of m modes are considered at the same time, the average normalized modal strain

energy change ratio for the jth element may also be utilized as the damage localization indicator, and it is defined as

$$MSECR_j = \frac{1}{m}\sum_{i=1}^{m}\frac{MSECR_{ij}}{MSECR_{i,\max}} \tag{3}$$

which is the average of MSECR normalized with respect to the largest value of $MSECR_{i,\max}$ in each ith mode.

It should be noted that the MSECR can also be represented by the ratio contributed by different sectional properties of the elements if the elemental stiffness matrix is considered as the combination of stiffness matrices contributed by different sectional properties (which will be disscussed later).

Neglecting the Elements with Small MSE

Because the elements with small MSE will inevitably lead to abnormal of MSECR value, especially in the application to a 3-D structure, criterion for eliminating the possibility of resulting in abnormal MSECR has been proposed [8] to neglect the jth element in i^{th} mode if

$$MSE_{ij} < C_{MSE}\times\frac{1}{L}\sum_{j=1}^{L}MSE_{ij} \tag{12}$$

where C_{MSE} is defined as the threshold of MSE. Another reason to eliminate the elements of small MSE is that the corresponding sensitivities of this kind of elements maybe too small and hence numerically leads to abnormal results of the inverse method. By setting the moderate C_{MSE} value, the null hypothesis of damage location and also the abnormal results of damage quantification can be removed. [8]

DAMAGE QUANTIFICATION

Original MSEC Method

The MSEC method [6] assumes that the damage only affects the stiffness matrix of the system, and the lump value of the stiffness loss of the jth element after the damage is introduced is expressed as

$$\Delta K_j = \alpha_j K_j \quad (-1 < \alpha_j \leq 0) \tag{4}$$

where α_j is the reduction factor in stiffness of jth element. The first order modal strain energy change of jth element in the ith mode due to the variation of mode shape is defined as

$$MSEC_{ij} = 2\Phi_i^T K_j \Delta\Phi_i = 2\Phi_i^T K_j(\Phi_i^d - \Phi_i) \tag{5}$$

If the variation of mode shape is assumed as the linear combination of the mode shapes, it can be derived from the equation of motion as [9]

$$\Delta\Phi_i = -\sum_{r=1}^{n} \frac{\Phi_r^T \Delta K \Phi_i}{\lambda_r - \lambda_i} \Phi_r \quad \text{where} \quad r \neq i \tag{6}$$

And substituting Eq. (6) into Eq. (5), $MSEC_{ij}$ can be written as

$$MSEC_{ij} = 2\Phi_i^T K_j \left(\sum_{r=1}^{n} \frac{\Phi_r^T \Delta K \Phi_i}{\lambda_r - \lambda_i} \Phi_r \right) \quad \text{where} \quad r \neq i \tag{7}$$

Finally, substituting Eq. (4) into Eq. (7), it is obtained

$$MSEC_{ij} = \sum_{p=1}^{L} -2\alpha_p \Phi_i^T K_j \left(\sum_{r=1}^{n} \frac{\Phi_r^T K_p \Phi_i}{\lambda_r - \lambda_i} \Phi_r \right) \quad \text{where} \quad r \neq i \tag{8}$$

Defining the sensitivity coefficient as

$$\beta_{jp} = -2\Phi_i^T K_j \left(\sum_{r=1}^{n} -\frac{\Phi_r^T K_p \Phi_i}{\lambda_r - \lambda_i} \Phi_r \right) \quad \text{where} \quad r \neq i, \quad j = 1,2,...,J, \quad p = 1,2,...,P \tag{9}$$

Eq. (9) can be expressed as the following form

$$\begin{Bmatrix} MSEC_{i1} \\ MSEC_{i2} \\ ... \\ MSEC_{iJ} \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1P} \\ \beta_{21} & \beta_{22} & ... & \beta_{2P} \\ ... & ... & ... & ... \\ \beta_{J1} & \beta_{J2} & ... & \beta_{JP} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ ... \\ \alpha_P \end{Bmatrix} \tag{10}$$

where J is the size of the group of selected elements for MSEC computation, which may include or may not include the suspected damaged elements with $J \geq P$, and P is the number of suspected damaged elements.

The term of left side of Eq. (10) is the modal strain energy change of all elements in the $i-th$ mode, which can be calculated from Eq. (5) by utilizing the identified mode shapes of the damaged and undamaged state from experimental data. The sensitivity coefficient β_{jp} can be calculated from Eq. (9) by utilizing the analytical modal information of the undamaged state. However, the direct solution of Eq. (10) would yield poor results due to the nature of the ill-posed problem, especially when the data contains noise. Therefore, in order to reduce the ill-posed problem, the number of suspected damaged elements should be appropriately determined in the previous damage localization stage, and it is recommended to include several modes when solving Eq. (10). When there are m modes to be utilized to estimate the damage, the number of equations of Eq. (10) will expand to dimension of $m \times J$.

Modified MSEC Method

In practice, there are no elements with equally reduction of stiffness in each DOF unless the element is totally removed or damaged. For a beam-column element, the stiffness directly relates to the sectional properties. Considering the elemental stiffness matrix of the jth element as the combination of stiffness matrices contributed by

different sectional properties, the variation of the stiffness matrix for j^{th} element can be expressed as

$$\Delta K_j = \alpha_j^A K_j^A + \alpha_j^{I_{33}} K_j^{I_{33}} + \alpha_j^{I_{22}} K_j^{I_{22}} + \alpha_j^{I_{11}} K_j^{I_{11}} \tag{11}$$

where superscript A denotes the one related to the cross sectional area, superscript I_{33} or I_{22} denotes the one related to the moment of inertia about the local 3rd axis or 2nd axis, respectively, and superscript I_{11} denotes the one related to the torsional constant. Therefore, the first order modal strain energy change of jth element in the ith mode due to the variation of mode shape and different sectional properties are expressed as

$$MSEC_{ij} = 2 \left\{ \begin{array}{l} \Phi_i^T K_j^A \Delta\Phi_i \\ \Phi_i^T K_j^{I_{33}} \Delta\Phi_i \\ \Phi_i^T K_j^{I_{22}} \Delta\Phi_i \\ \Phi_i^T K_j^{I_{11}} \Delta\Phi_i \end{array} \right\} \tag{12}$$

And the sensitivity coefficient is modified as

$$\beta_{jp} = \begin{bmatrix} \beta_{jp}^{AA} & \beta_{jp}^{AI_{33}} & \beta_{jp}^{AI_{22}} & \beta_{jp}^{AI_{11}} \\ \beta_{jp}^{I_{33}A} & \beta_{jp}^{I_{33}I_{33}} & \beta_{jp}^{I_{33}I_{22}} & \beta_{jp}^{I_{33}I_{11}} \\ \beta_{jp}^{I_{22}A} & \beta_{jp}^{I_{22}I_{33}} & \beta_{jp}^{I_{22}I_{22}} & \beta_{jp}^{I_{22}I_{11}} \\ \beta_{jp}^{I_{11}A} & \beta_{jp}^{I_{11}I_{33}} & \beta_{jp}^{I_{11}I_{22}} & \beta_{jp}^{I_{11}I_{11}} \end{bmatrix} \tag{13}$$

in which the typical component of sensitivity coefficient β_{jp} is expressed as

$$\beta_{jp}^{AI_{33}} = -2\Phi_i^T K_j^A \left(\sum_{r=1}^{n} -\frac{\Phi_r^T K_p^{I_{33}} \Phi_i}{\lambda_r - \lambda_i} \Phi_r \right) \quad \text{where } r \neq i \tag{14}$$

where the combination of superscript A or I_{33} can be replaced by any other combination of A, I_{33}, I_{22}, and J.

Because the natural frequencies are determined much more accurately than the mode shapes, and the incoroporation of the change of system natural frequency and MSEC may also reduce the posibiliy of ill-posed sensitivity matrix, and the sensitivity equations of the variation of natural frequencies is:

$$\Delta f_i = f_i^d - f_i^0 = \alpha_i \frac{\Phi_i^T K_j \Phi_i}{8\pi^2 f_i} \tag{15}$$

are added to Eq. (10), where f_i^d is the measured natural frequency of the ith mode of the damage system, and f_i^0 is the measured intact natural frequency of the ith mode, hence Eq. (10) turns into

$$\left\{ \begin{array}{c} \Delta f_i \\ MSEC_{i1} \\ MSEC_{i2} \\ ... \\ MSEC_{iJ} \end{array} \right\} = \left[\begin{array}{cccc} \dfrac{\Phi_i^T K_1 \Phi_i}{8\pi^2 f_i} & \dfrac{\Phi_i^T K_2 \Phi_i}{8\pi^2 f_i} & ... & \dfrac{\Phi_i^T K_J \Phi_i}{8\pi^2 f_i} \\ \beta_{11} & \beta_{12} & ... & \beta_{1P} \\ \beta_{21} & \beta_{22} & ... & \beta_{2P} \\ ... & ... & ... & ... \\ \beta_{J1} & \beta_{J2} & ... & \beta_{JP} \end{array} \right] \left\{ \begin{array}{c} \alpha_1 \\ \alpha_2 \\ ... \\ \alpha_P \end{array} \right\} \tag{16}$$

Finally, Eq. (12), Eq. (13) and Eq. (14) are substituted into Eq. (16) to solve for the stiffness reduction factor of different sectional property of each element.

Dynamic Modal Expansion

The dynamic expansion is a well known reduction and expansion method based on the undamped dynamic equation of the system. [10] The unmeasured DOFs of the current structure state can be obtained by expanding the measured DOFs of the structure based on the following equation

$$\Phi_s = -(K - M\omega_i^2)_{ss}^{-1}(K - M\omega_i^2)_{sm}\Phi_m \tag{11}$$

where subscript m relates to master DOFs (measured DOFs), and subscript s relates to slave DOFs (un-measured DOFs). The mode shape with full DOFs can be obtained by expanding the incomplete measured mode shape using this dynamic modal expansion algorithm.

Modified Iteration Process

Because the relation between the MSEC and the damage reduction factor α_j is nonlinear when enough damage has occurred to cause the shift of the system natural frequency and stiffness, iteration process is required to obtain more accurate assessment of damage severity. Ricles and Kosmatka had proposed an iteration process coping with the nonlinearity [11] as illustrated in Fig. 1(a), where the superscripts 0, 1, and 2 refer to the linearization points during updating, and Λ_0 and Λ_d are arrays containing modal parameters including natural frequency and MSE of the intact and damaged structure, respectively. The difference between Λ_0 and Λ_d is actually the left side of Eq.(16). In the ith step, the stiffness reduction factor array α^i is calculated by Eq. (16) which considering the current structural state rather than the intact structural state. Finally, the accumulated stiffness reduction factor array α is obtained by summing all the stiffness reduction factor array α^i.

The target of the iteration process is Λ_d, which is assumed fixed during the iteration. The natural frequency in Λ_d is directly the measured one in the damaged state. However, the other target, MSE in the damaged state, is obtained by dynamic expanding the measured mode shape in the damaged state according to the stiffness matrix. In the original iteration process, the intact stiffness matrix is utilized and remains unchanged during the iteration. We propose to use the stiffness matrix updated based on the results of the previous step in each iteration step to expand the measured

mode shape, hence the updated target MSE is utilized during the iteration process. The modified iteration process is demonstrated in Fig. 1(b).
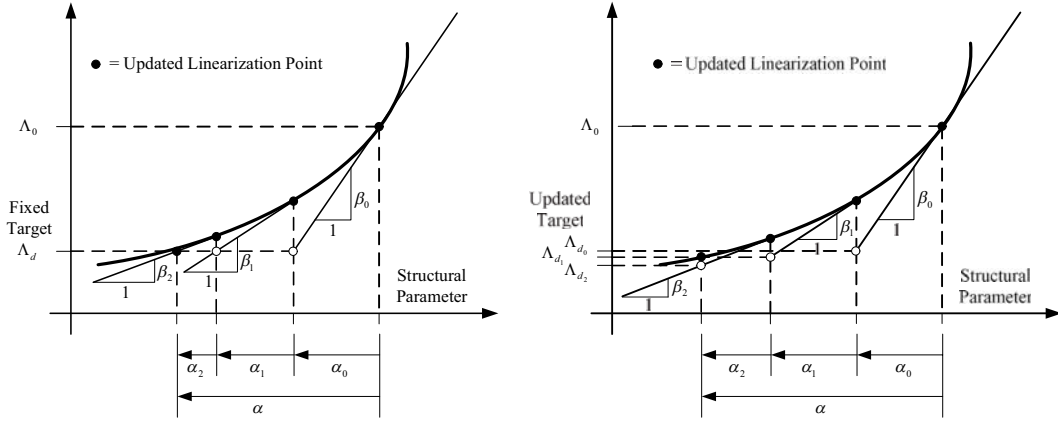

Fig. 1 Iteration process (a) original; (b) modified

Convergence Criterion

In practice, the true damage state is unknown. Therefore, certain criterion is required to evaluate the results obtained by the modified MSEC method. Actually, both the natural frequency, mode shape, and stiffness matrix must satisfy the equation of motion, i.e. $(K - M\omega_i^2)\phi_i = 0$, but the residual is not a scalar. In this study, we use the measured damaged natural frequency as the target, and define the target ratio (TR) as the convergence criterion

$$TR_i = \frac{f_i^j - f_i^d}{f_i^o - f_i^d} \tag{11}$$

where f_i^j is the calculated natural frequency of the ith mode in the jth iteration. The advantage of using natural frequency based convergence criterion is that the natural frequency is measured directly and also reliable. This simple convergence criterion provides a roughly idea about the reliability of the results.

EXPERIMENTAL SETUP AND FEM OF THE TEST STRUCTURE

Experimental Setup of the Test Structure

The modified MSEC procedure is evaluated using modal data extracted from a shaking table test of a 3D structure, which is a full-scale 1-bay × 1-bay × 3-story steel frame structure (Fig. 2). The dimension of the test structure is 2m, 3m and 9m in X, Y and Z direction, respectively. The dead load is simulated by lead-block units fixed on the steel plate of each floor, results in the total mass of each floor of the test structure is 5,943 kg. To imitate the damage state which is like the plastic hinge of the column, the flanges of

343

the bottom of the first story column are sliced with 2cm wide and 20cm long for both sides. According to the numerical study of the entire damaged column of the first floor by SAP2000 software, the force need to achieve 1 unit deformation in each DOF on the top of the designated-damaged column (i.e. the point No. 5 and 6 in Fig. 3) is deducted to different extent which is summarized in Table 2.

Fig. 2 Photo of the 3-D experimental test structure

Table 2: Stiffness reduction of each DOF on the top of the sliced column

Global Coordinate	Reduced Quantity
Z	-2.5%
X	-20.3%
Y	-5.5%
RZ	-3.1%
RX	-6.8%
RY	-25.2%

The test structure was subjected to El Centro earthquakes and random vibration simulated by the shaking table in National Center of Research on Earthquake Engineering (NCREE), Taiwan, R.O.C.. Unilateral, bilateral, and torsional excitation from shaking table test with amplitude 100 gal are conducted both before and after the "damage" is introduced into the test structure. The acceleration responses during the tests are measured only at point 6, 8, 11, 13, 16, and 18 in the X direction and point 7, 8, 12, 13, 17, and 18 in the Y direction.

To give an outline of the limited number of measured DOFs of the test structure in this study, the Coefficient of Measurement Density (CMD) is defined as (measured number of DOFs)/((number of elements)×(number of DOFs per node)). The CMD of the test structure is (12)/(36×6)=1/18, on the other hand, the CMD of the experimental case study of the original paper [6] is (16)/(18×3)=8/27, which is more than 5 times of the CMD of the test structure. Accordingly, the hardship for damage detection of this test structure can be anticipated.

FEM of Test Structure

The FEM of test structure is simplified and condensed into 36 beam-column elements. Each of the point has 6 DOFs, therefore there are 90 DOFs totally. The axial stiffness of the elements No. 9~12, 21~24, 33~36 are magnified to simulate the stiffness contribution of the steel plate and lead block units. The distribution of the mass of each joint in each floor is manually adjusted to fit the experimental modal results. The details of the geometrical and physical information of the test structure are shown in Fig.3.
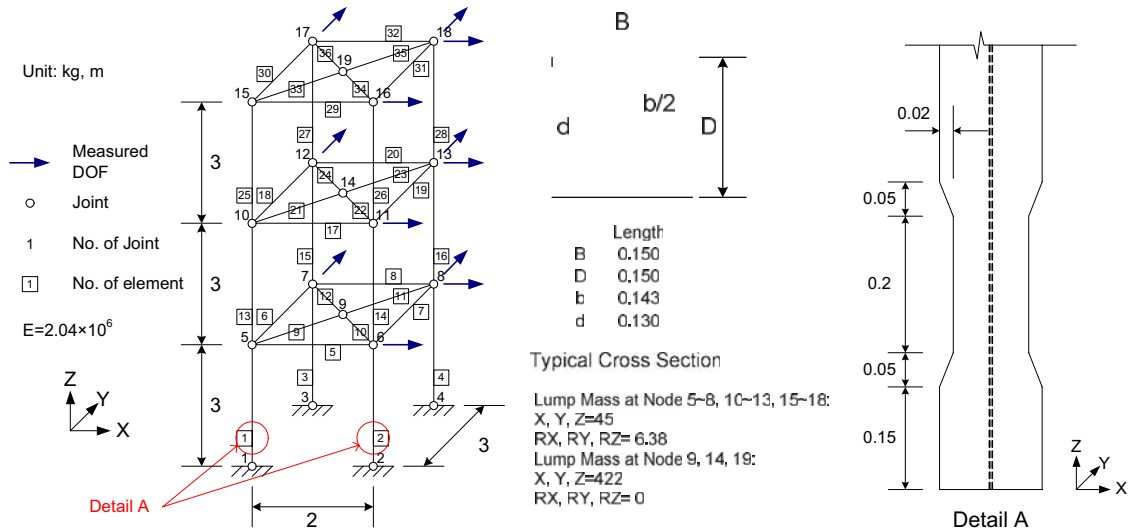
Fig. 3 FEM of the 3D test structure

The modal frequencies and mode shapes are determined by the Frequency Domain Decomposition (FDD) technique [12] from the experimental data, and then the incomplete mode shapes are expanded by utilizing dynamic expansion algorithm. The modal frequency of the intact test structure obtained utilizing FDD technique are summarized in Table 3, and the analytical results of the FEM are also compared in the same table. The first 3 measured mode shapes and the analytical one are drawn in Fig. 4. In order to evaluate the measured mode shape extracted from the experimental data, the Modal Assurance Criterion (MAC) is utilized to give a rough idea. The diagonal values of the MAC between the experimental and analytical mode shapes are also listed in Table3. In summary, the differences between the analytical and experimental natural frequencies are all less than 5%, and the diagonal values of the MAC are all larger than 0.99. It is concluded that the FEM of the test structure is capable to represent the real test structure. The damaged experimental mode shapes are also identified from the test data (structure with reduce cross section at the first floor columns), and are compared with the intact and damaged experimental mode shapes, as shown in Fig. 5. The differences of the mode shapes are quite small visually.

To imitate the true damage state of the test structure, the sectional properties of the 1st and 2nd element are reduced similar to the results in Table 2, which is summarized in Table 4. The approximate stiffness reduction of sectional properties of element 1 and 2 will be utilized in the following numerical study.

NUMERICAL STUDY OF THE TEST STRUCTURE

In order to verify the modified iteration process and the modified MSEC method, the numerical study of the test structure is performed. Due to the very limited measurements of the test structure, the numerical study of the effect caused by modal expansion is also performed in advance. As a result, the numerical study involves 2

phases, and each phase contains 3 cases, which are summarized in Table 5. In both phases and the later experimental study, the first 78 analytical modes without considering noise are utilized in the numerical and experimental study of the test structure.
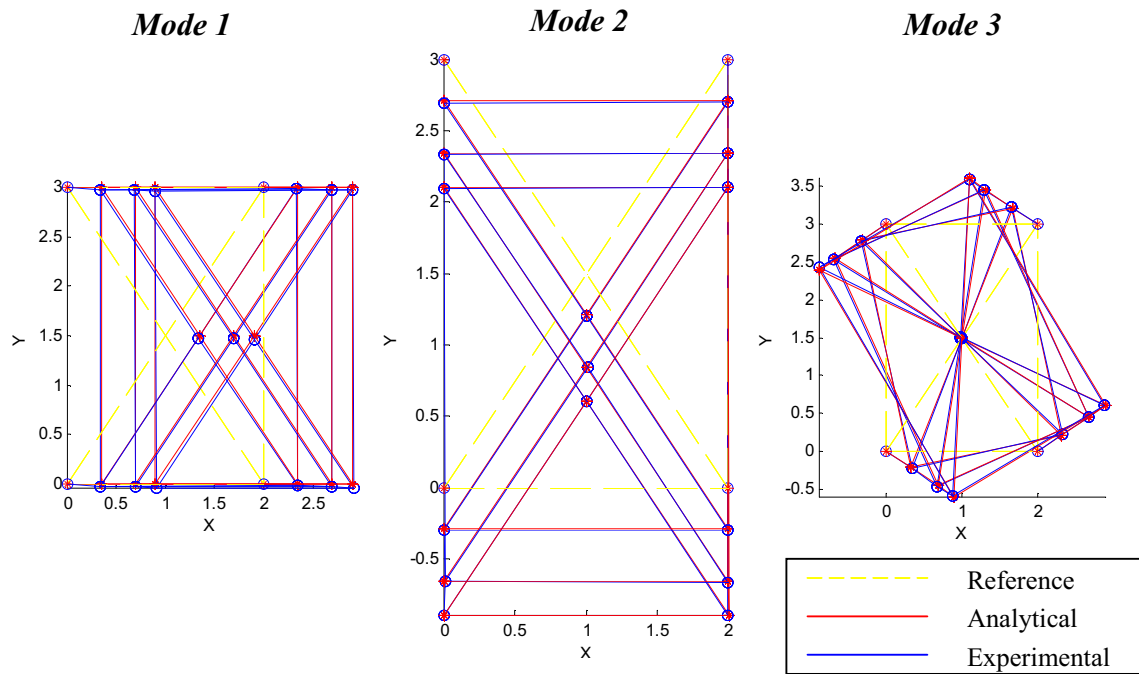

Fig. 4 Top view of the analytical and experimental mode shapes of the intact test structure
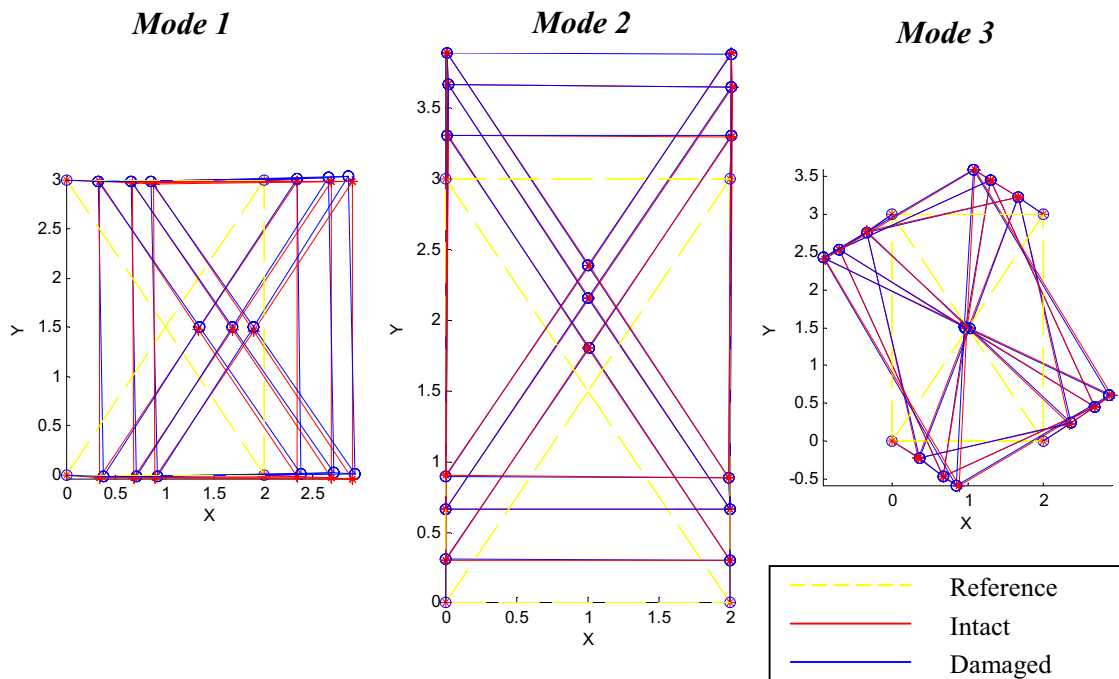

Fig. 5 Top view of the experimental mode shapes of the intact and damaged test structure

Table 3 Comparing between analytical and experimental natural frequencies of the intact test structure

Mode	Analytical Freq. (Hz)	Experimental Freq. (Hz)	Error (%)	MAC
X1	1.121	1.128	-0.6%	99.81%
Y1	1.458	1.389	4.9%	99.98%
T1	2.139	2.083	2.7%	99.91%
X2	3.311	3.299	0.4%	99.86%
Y2	4.722	4.601	2.6%	99.71%
X3	5.122	5.165	-0.8%	99.86%
T2	6.539	6.489	0.8%	99.45%
Y3	8.219	8.181	0.5%	99.76%
T3	10.645	10.81	-1.5%	99.50%

Table 4 Approximate stiffness reduction of sectional properties of element 1 and 2 in the numerical study

Sectional Property	Reduced Quantity
A	-2.0%
I_{33}	-20.0%
I_{22}	-6.0%
I_{11}	-3.0%

Table 5 Summary of the numerical study

		Phase 1 (Damage Localization)			Phase 2 (Damage Quantification)		
		Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
Mode Shapes	Analytical	Y	Y	-	-	Y	
	Dynamic Expanded	-	-	Y	Y	-	Y
MSEC	Original	Y		-	-	Y	-
	Modified	-	Y	Y	Y	Y	Y
Iteration Process	Original	-	-	-	Y	-	-
	Modified	-	-	-	Y	Y	Y

1st Phase: Damage Localization

The purpose of the numerical study of the 1st phase is to identify the suspected damage elements by using MSECR under different situations. The elements/ sectional-properties of each mode with MSECR grater than 0.05 are selected as the suspected damaged ones which will be utilized to calculate the stiffness reduction factors in the following section. Take the results of the 1st mode (analytical mode shape) in case 1 for example (Fig. 7(a)), there are only 2 elements, i.e. element 1 and 2, whose

MSECR are greater than 0.05. Therefore, only these 2 elements are selected as the suspected damaged elements and utilized to calculate the stiffness reduction factors for the 1st mode. The difference between case 1 and case 2 is the calculation of MSECR. The other example is the results obtained by the 1st mode in case 3 (Fig. 7(c)). There are 3 sectional-properties whose MSECR exceed the limit, i.e. I_{33} of the element 1, I_{33} of the element 2, and the I_{22} of the element 5.

In order to simplify the comparison of the results obtained by different methods in the numerical study of the 2nd phase, the suspected damaged elements/sectional -properties are selected considering the same mode numbers, rather than the individual mode. For example, the 1st, 4th and 6th mode are the 1st, 2nd and 3rd mode of the X-direction, respectively, and the average MSECR of these 3 modes is considered to locate the suspected damage elements/sectional-properties. Similarly, the 2nd, 5th, and 8th modes are considered together, and the 3rd, 7th, and 9th modes are considered together.

Observing these results in Fig. 6, it is found that the lower modes can clearly provide information to indicate the damaged elements, while the higher modes can not. Therefore, when coping with the experimental study of the test structure, the information of damage location from higher modes should not be considered in order to identify the damage elements clearly. Of course this conclusion is directly related to the type of scenario damage. The other conclusion may be made that different kinds of mode shapes (e.g. X-dir., Y-dir., and Torsion) are relate to different kind of element damage (e.g. I_{33}, I_{22}, I_{11}, and A).

2nd Phase: Damage Quantification

The numerical study in the first 2 cases in the 2nd phase is to evaluate the proposed methods by comparing the results obtained by the proposed method to the results obtained by the original method. While in case 3 of the 2nd phase illustrates the effects caused by the dynamic expanded mode shape. Case 3 is also the most similar one to the experimental study, which maybe a good example before the experimental study is performed.

Case 1:

In case 1, comparison on the stiffness reduction factors obtained by the original and modified iteration process is made. The complete mode shapes are obtained by dynamic expanding of the measured responses from 12 DOFs (the same as the measured 12 DOFs in the experimental study), and the proposed modified MSEC is conducted to calculate the stiffness reduction factors. Here the performance of the proposed modified iteration process by using the suspected damaged elements selected in the 1st mode of the 1st phase numerical study, i.e. the 1st bar chart is demonstrated in Fig. 6(c). The stiffness reduction factors obtained by the original and modified iteration process at each step is illustrated in Fig. 7(a) and 7(b), respectively. The original iteration process converges very quickly but the results are relatively bad, i.e. the sectional property I_{22} of element 5 should be no damage, but the result is

approximately 10% reduction. On the other hand, the results of modified iteration process converge to the real reduction value after around 30 iterations. The modified iteration process which updated the target modal parameters considering the current model state is proven to be a much better iteration process.

(a)
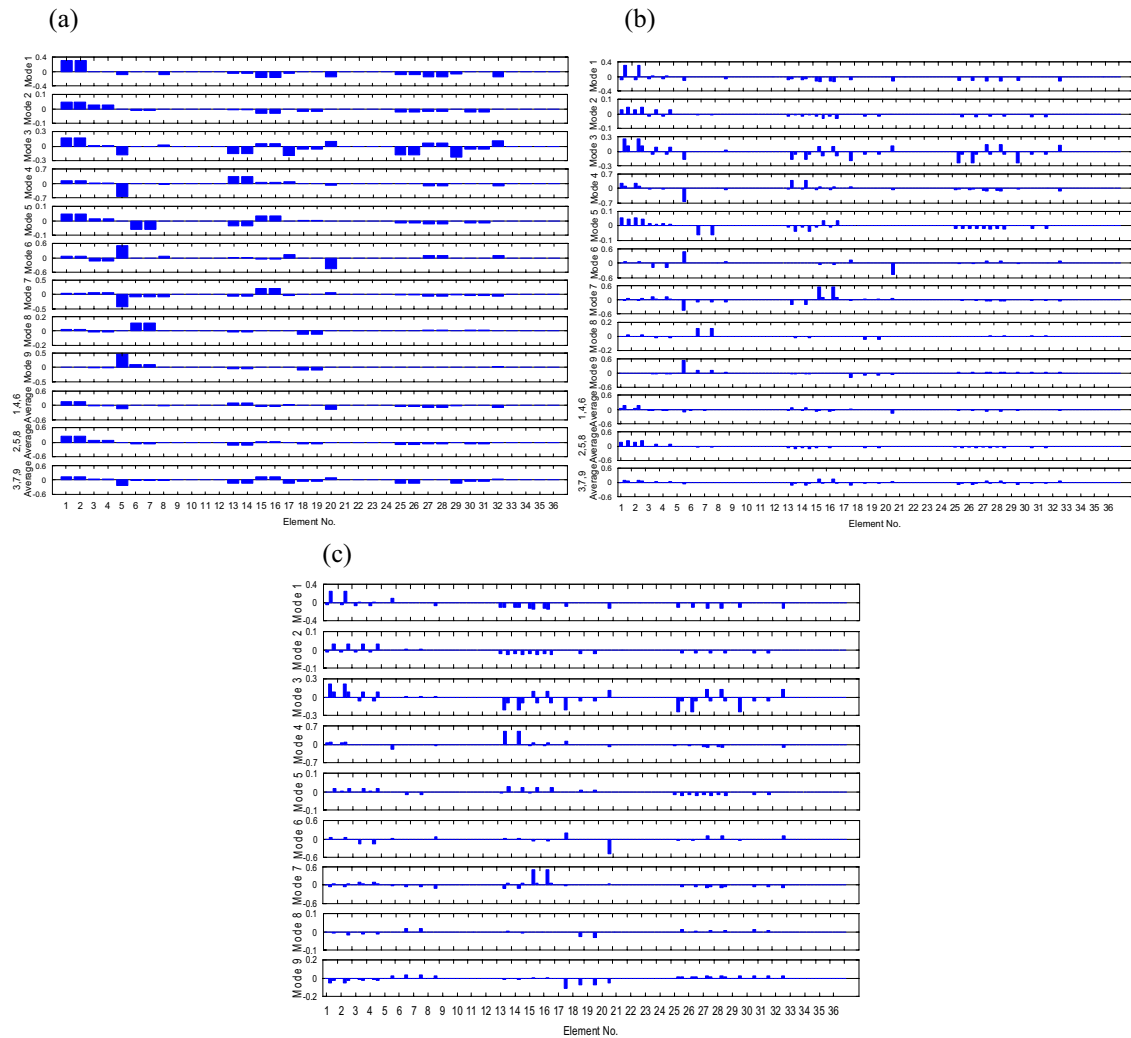
(b)

(c)

Fig. 6 Damage localization results of the 3D test structure in the 1st phase numerical study (a) Case 1, (b) Case 2, and (c) Case 3
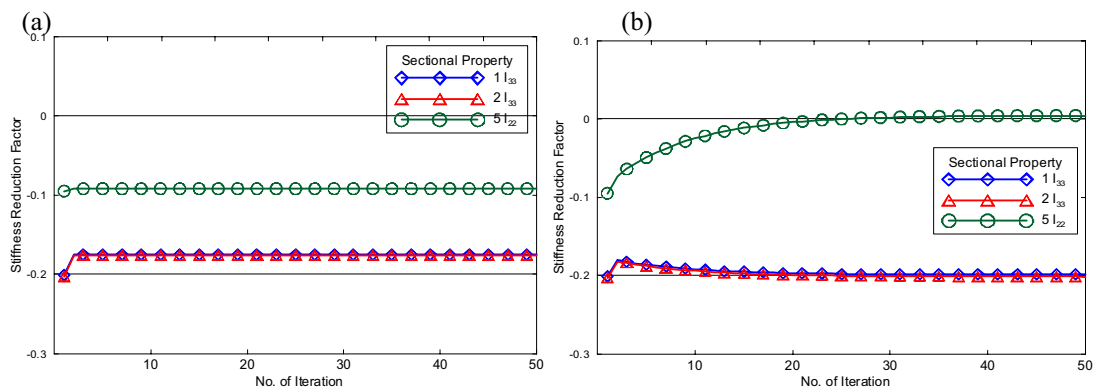
(a)

(b)

Fig. 7 Damage quantification results of (a) original, (b) modified, iteration process.

Case 2:

In order to demonstrate the effectiveness of the modified MSEC method, the stiffness reduction factors obtained by the original and modified MSEC methods are compared in case 2 study by using the modified iteration method and also using the analytical complete mode shapes. Since each individual mode can be used to calculate the stiffness reduction factors, but for simplification only shows the comparison obtained by different combination of modes. Five kinds of combination in case 2 is studied, i.e. (1) mode 1, 4, and 6; (2) mode 2, 5, and 8; (3) mode 3, 7, and 9; (4) mode 1, 2, 4, 5, 6, 8; (5) mode 1through 9. The results obtained by the original and modified MSEC are summarized in Table 6 and Table 7, respectively. Only 3 iterations are performed since the analytical complete mode shapes are used.

In Table 6, the stiffness reduction factors obtained by the combination of 3 X-directional modes are quite well since it relates to the reduction of I_{33} of the 1st and 2nd element, which is approximately 20% reduction. The target ratios of the corresponding modes of the X-directional modes are close to zero, while the target ratios of the other modes are not. Similarly, the stiffness reduction factors obtained by the combination of 3 Y-directional modes are also quite well. However, the stiffness reduction factors of the 1st and 2nd elements obtained by the combination of 3 torsional modes are around the average of 20% reduction and 6% reduction, and the stiffness reduction factors of the 33th element is quite wrong. The combination of the 6 X- and Y-directional modes or the combination of all the 9 modes face the same situation, and the stiffness reduction factors of the 1st and 2nd elements are quite different from the one obtained by the torsional mode shapes. Because the results obtained by the original MSEC method are actually the "lump sum" stiffness reduction of the elements, which is not the true damage state in the numerical study, the target ratios obtained by the last 3 kinds of combination does not give a clear idea of whether the results are reliable or not.

On the other hand, in Table 7, the stiffness reduction factors of the 1st and 2nd elements obtained by the combination of X- or Y-directional modes identify not only the corresponding sectional properties I_{33} or I_{22} respectively, but also the axial sectional property A. The stiffness reduction factors of the 1st and 2nd elements obtained by the last 3 kind of combinations of modes are almost the same as the assigned damage extent, and the corresponding target ratios of these 3 kinds of combination distinctly indicate that the results are reliable since the natural frequency obtained by the calculated damaged stiffness matrix is nearly the same as the measured natural frequencies in the damaged state.

Case 3:

From case 1 and case 2 it have proved the modified MSEC method and modified iteration process perform better than the original methods. In case 3, the effect caused by the expansion under such few measurements by using the modified MSEC and modified iteration process with dynamic expanded mode shapes. The results obtained

Table 6 Results of original MSEC method (analytical complete mode shapes)

Mode 1,4,6				Mode 2,5,8				Mode 3,7,9				Mode 1,2,4,5,6,8				Mode 1~9		
Element No.	α	Real α		Element No.	α	Real α		Element No.	α	Real α		Element No.	α	Real α		Element No.	α	Real α
1	-19.2%	-20.0%		1	-6.0%	-6.0%		1	-11.4%	-		1	-7.5%	-		1	-9.1%	-
2	-19.2%	-20.0%		2	-6.0%	-6.0%		2	-11.3%	-		2	-7.5%	-		2	-9.1%	-
13	-0.1%	0.0%		3	0.0%	0.0%		16	0.1%	0.0%		3	1.2%	0.0%		3	-1.3%	0.0%
14	-0.1%	0.0%		4	0.0%	0.0%		17	-2.2%	0.0%		4	1.2%	0.0%		4	-1.3%	0.0%
								33	36.0%	0.0%		13	-0.7%	0.0%		13	-0.1%	0.0%
												14	-0.7%	0.0%		14	-0.1%	0.0%

Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio
1	X	1.0%		1	X	-16.4%		1	X	-21.0%		1	X	-33.3%		1	X	-24.5%
2	Y	24.6%		2	Y	0.0%		2	Y	9.6%		2	Y	1.5%		2	Y	8.0%
3	T	-45.2%		3	T	-17.9%		3	T	-23.3%		3	T	57.5%		3	T	30.1%
4	X	-0.5%		4	X	-40.0%		4	X	-42.4%		4	X	-70.4%		4	X	-50.0%
5	Y	87.6%		5	Y	0.0%		5	Y	33.6%		5	Y	3.1%		5	Y	27.6%
6	X	-0.6%		6	X	-26.5%		6	X	-29.1%		6	X	-38.0%		6	X	-31.8%
7	T	35.3%		7	T	-45.1%		7	T	-4.1%		7	T	-32.6%		7	T	-12.0%
8	Y	109.2%		8	Y	0.0%		8	Y	41.7%		8	Y	12.2%		8	Y	35.1%
9	T	51.2%		9	T	-39.7%		9	T	0.2%		9	T	-20.9%		9	T	-5.0%

Table 7 Results of modified MSEC method (analytical complete mode shapes)

Mode 1,4,6				Mode 2,5,8				Mode 3,7,9				Mode 1,2,4,5,6,8				Mode 1~9		
Sectional Property	α	Real α		Sectional Property	α	Real α		Sectional Property	α	Real α		Sectional Property	α	Real α		Sectional Property	α	Real α
'1_A'	-2.3%	-2.0%		'1_A'	-1.9%	-2.0%		'1_Iz'	-19.9%	-20.0%		'1_A'	-2.0%	-2.0%		'1_A'	-1.4%	-2.0%
'1_Iz'	-19.7%	-20.0%		'1_Iy'	-6.0%	-6.0%		'1_Iy'	-6.0%	-6.0%		'1_Iz'	-19.7%	-20.0%		'1_Iz'	-19.9%	-20.0%
'2_A'	-2.3%	-2.0%		'2_A'	-1.9%	-2.0%		'2_Iz'	-19.9%	-20.0%		'1_Iy'	-6.0%	-6.0%		'1_Iy'	-6.0%	-6.0%
'2_Iz'	-19.7%	-20.0%		'2_Iy'	-6.0%	-6.0%		'2_Iy'	-6.0%	-6.0%		'2_A'	-2.0%	-2.0%		'2_A'	-1.4%	-2.0%
'13_Iz'	0.0%	0.0%		'3_Iy'	0.0%	0.0%		'15_Iz'	0.0%	0.0%		'2_Iz'	-19.7%	-20.0%		'2_Iz'	-19.9%	-20.0%
'14_Iz'	0.0%	0.0%		'4_Iy'	0.0%	0.0%		'16_Iz'	0.0%	0.0%		'2_Iy'	-6.0%	-6.0%		'2_Iy'	-6.0%	-6.0%
												'3_Iy'	0.0%	0.0%		'3_Iy'	0.0%	0.0%
												'4_Iy'	0.0%	0.0%		'4_Iy'	0.0%	0.0%
												'13_Iz'	0.0%	0.0%		'13_Iz'	0.0%	0.0%
												'14_Iz'	0.0%	0.0%		'14_Iz'	0.0%	0.0%

Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio
1	X	0.2%		1	X	-16.4%		1	X	4.4%		1	X	0.0%		1	X	-0.2%
2	Y	-3.8%		2	Y	0.0%		2	Y	-1.4%		2	Y	0.0%		2	Y	0.0%
3	T	-24.4%		3	T	-17.9%		3	T	0.8%		3	T	0.2%		3	T	0.1%
4	X	0.4%		4	X	-40.0%		4	X	3.0%		4	X	0.0%		4	X	-0.7%
5	Y	-14.6%		5	Y	0.0%		5	Y	-1.6%		5	Y	0.0%		5	Y	0.1%
6	X	-1.7%		6	X	-26.5%		6	X	-1.1%		6	X	0.0%		6	X	-1.9%
7	T	-55.1%		7	T	-45.1%		7	T	0.7%		7	T	0.4%		7	T	0.0%
8	Y	-19.3%		8	Y	0.0%		8	Y	-0.2%		8	Y	0.0%		8	Y	-0.1%
9	T	-42.6%		9	T	-39.7%		9	T	0.6%		9	T	0.3%		9	T	-0.6%

by individual 1st, 2nd, 3rd, and 4th modes are summarized in Table 8, while the other results obtained by individual 5th through 9th modes diverge and hence are not shown. It proves again that higher modes are too sensitive to be expanded and great error is introduced. The iteration process is terminated if the summation of the variation of target ratios of all the 9 modes is less than 0.001, and the iteration number of each mode is also listed in Table 8. The results in Table 8 show that the stiffness reduction of sectional properties I_{33} of element 1 and 2 are correctly identified by the first 2 X-direction modes and the 1st torsional mode, while the stiffness reduction of sectional properties I_{22} of element 1 and 2 are distributed to I_{22} of element 3 and 4 in the first Y-direction mode. The stiffness reduction of sectional properties A of element 1 and 2 are incorrect because there are no vertical DOFs measured. Based on this case study, the following experimental study will use the same individual mode to identify the damage of the real test structure.

Table 8 Results of modified MSEC method (dynamic expanded mode shapes)

Mode 1 (ITE=30)				Mode 2 (ITE=21)				Mode 3 (ITE=6)				Mode 4 (ITE=12)		
Sectional Property	α	Real α		Sectional Property	α	Real α		Sectional Property	α	Real α		Sectional Property	α	Real α
'1_Iz'	-19.9%	-2.0%		'1_Iy'	-3.0%	-6.0%		'1_Iz'	-20.5%	-20.0%		'1_A'	20.5%	-2.0%
'2_Iz'	-20.1%	-20.0%		'2_Iy'	-3.0%	-6.0%		'1_Iy'	-3.0%	-6.0%		'1_Iz'	-18.5%	-20.0%
'5_Iy'	0.2%	-2.0%		'3_Iy'	-3.0%	0.0%		'2_Iz'	-20.7%	-20.0%		'2_A'	20.5%	-2.0%
				'4_Iy'	-3.0%	0.0%		'2_Iy'	-3.1%	-6.0%		'2_Iz'	-19.1%	-20.0%
				'6_Iy'	-0.1%	0.0%		'3_Iz'	-2.9%	0.0%		'13_Iz'	3.7%	0.0%
				'7_Iy'	-0.1%	0.0%		'3_Iy'	-2.8%	0.0%		'14_Iz'	2.4%	0.0%
								'4_Iz'	0.0%	0.0%		'15_Iz'	1.9%	0.0%
								'4_Iy'	0.1%	0.0%		'16_Iz'	2.0%	0.0%
								'5_Iy'	0.0%	0.0%		'17_Iy'	0.5%	0.0%
								'6_Iy'	-0.3%	0.0%				
								'7_Iy'	-0.4%	0.0%				
								'8_Iy'	3.2%	0.0%				

Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio		Mode No.	Dir.	Target Ratio
1	X	-0.2%		1	X	-16.4%		1	X	115.5%		1	X	-2.2%
2	Y	-3.9%		2	Y	0.1%		2	Y	-62.5%		2	Y	-2.9%
3	T	-68.8%		3	T	-17.9%		3	T	0.0%		3	T	161.0%
4	X	-0.6%		4	X	-40.0%		4	X	61.0%		4	X	0.0%
5	Y	-14.6%		5	Y	-0.5%		5	Y	-27.2%		5	Y	-13.8%
6	X	-1.2%		6	X	-26.5%		6	X	-925.2%		6	X	-0.6%
7	T	-48.2%		7	T	-42.7%		7	T	70.3%		7	T	-88.8%
8	Y	-19.3%		8	Y	-0.6%		8	Y	136.8%		8	Y	-19.3%
9	T	-44.6%		9	T	-36.3%		9	T	-245.8%		9	T	-17.6%

EXPERIMENTAL DAMAGE DETECTION

Damage Localization

The measured mode shapes of the intact and damaged structure are utilized for damage localization. Due to the noise and other disturbance in the test, the threshold of MSE C_{MSE} is chosen as 0.3 in the experimental study. Similar to the results of numerical study (Fig. 6(c)), the damage locations can be successfully identified by only the first 3 modes. It should be noted that the MSECR value of the 4th mode can not identify the damage locations, which implies the damage quantification obtained by the 4th mode may not be as good as the numerical study. The sectional properties with MSECR greater than 0.05 of the first 4 individual mode are chosen to calculate the stiffness reduction factors.

Damage Quantification

The measured mode shapes and natural frequencies of the intact and damaged structure are utilized for damage quantification. Based on the results of numerical study, the stiffness reduction factors obtained using the first 4 individual modes are listed in Table 9. Because the iteration process fails to converge for these 4 modes, only the results obtained at the first iteration are shown. From the results, it seems that the stiffness reduction of sectional properties I_{33} of element 1 and 2 are properly

identified by the first 2 X-direction modes and the 1st torsional mode, and the target ratios of the corresponding modes are nearly zero. However, the other sectional properties of other elements are identified as some moderate amount of stiffness

reduction or "increasing", which may be caused by modal expansion, modeling error, and noise effect et al. The combination of any modes does not improve the results of damage quantification and hence not shown here.

As discussed previously, the number of measured DOFs is relative sparse which leads to great error caused by expansion of incomplete mode shapes. The modeling error is also a tough question of the model based identification method. Although the MAC value and cross-orthogonality check (COR, which is not shown in this paper) of the measured and analytical mode shapes are quite excellent, it seems that the modeling error still introduce great error to the modified MSEC method. Further study is needed to identify the effect of modeling error. In this study, certain amount of noise should introduce moderate level of errors for damage quantification since the MSEC method has been proved noise sensitive in damage quantification [6].
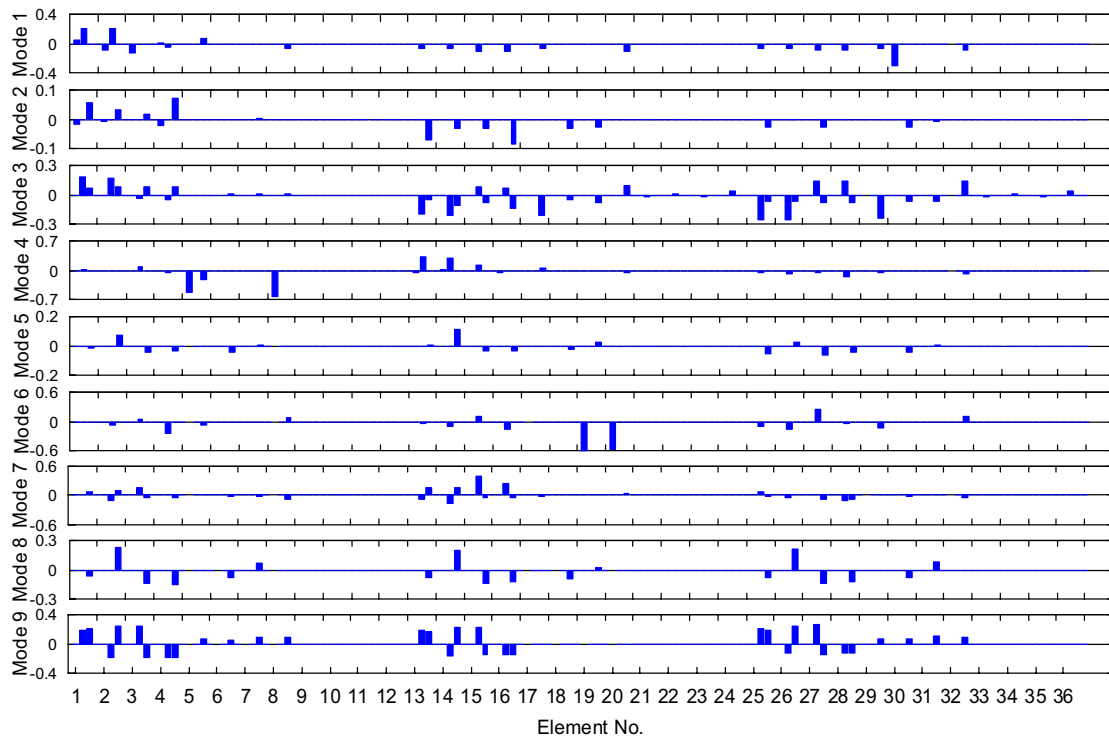

Fig. 8 Experimental damage localization results of the test structure

CONCLUSIONS

The modified MSEC method to detect structural damage and corresponding modified iteration process are proposed in this paper. Numerical results using a 3-D frame structure clearly illustrate the following:

(1) The reduction of sectional properties rather than lump stiffness of elements can be identified by the proposed modified MSEC method, hence the torsional modes and the combination of different kind of modes become capable to identify the correct damage extent.

Table 9 Results of experimental damage detection

Mode 1 (ITE=1) Sectional Property	α	Analytic α	Mode 2 (ITE=1) Sectional Property	α	Analytic α	Mode 3 (ITE=1) Sectional Property	α	Analytic α	Mode 4 (ITE=1) Sectional Property	α	Analytic α
'1_A'	-2.4%	-2.5%	'1_Iy'	-14.2%	-5.5%	'1_Iz'	-22.2%	-20.3%	'1_Iz'	-22.3%	-20.3%
'1_Iz'	-16.5%	-20.3%	'2_Iy'	-5.9%	-5.5%	'1_Iy'	3.5%	-5.5%	'2_Iz'	-24.1%	-20.3%
'2_Iz'	-17.4%	-20.3%	'3_Iy'	0.1%	0.0%	'2_Iz'	-21.1%	-20.3%	'13_Iz'	20.0%	0.0%
'4_A'	0.6%	0.0%	'4_Iy'	-9.9%	0.0%	'2_Iy'	4.4%	-5.5%	'14_Iz'	16.7%	0.0%
'5_Iy'	-8.8%	0.0%				'3_Iy'	0.1%	0.0%	'15_Iz'	-12.5%	0.0%
						'4_Iy'	1.1%	0.0%	'16_Iz'	-11.1%	0.0%
						'15_Iz'	6.9%	0.0%	'17_Iy'	5.2%	0.0%
						'16_Iz'	6.5%	0.0%			
						'18_A'	-2.0%	0.0%			
						'19_A'	-2.8%	0.0%			
						'20_Iy'	6.9%	0.0%			
						'27_Iz'	6.9%	0.0%			
						'28_Iz'	7.1%	0.0%			
						'29_A'	-4.5%	0.0%			
						'30_A'	-9.4%	0.0%			
						'31_A'	-7.1%	0.0%			
						'32_A'	-1.5%	0.0%			
						'32_Iy'	12.5%	0.0%			

Mode No.	Dir.	Target Ratio	Mode No.	Dir.	Target Ratio	Mode No.	Dir.	Target Ratio	Mode No.	Dir.	Target Ratio
1 (X)	X	-0.2%	1	X	-73.8%	1	X	115.5%	1	X	-2.2%
2 (Y)	Y	-3.9%	2	Y	-362.2%	2	Y	-62.5%	2	Y	-2.9%
3 (T)	T	-68.8%	3	T	-327.6%	3	T	0.0%	3	T	161.0%
4 (X)	X	-0.6%	4	X	-29.6%	4	X	61.0%	4	X	0.0%
5 (Y)	Y	-14.6%	5	Y	-342.7%	5	Y	-27.2%	5	Y	-13.8%
6 (X)	X	-1.2%	6	X	748.5%	6	X	-925.2%	6	X	-0.6%
7 (T)	T	-48.2%	7	T	-142.2%	7	T	70.3%	7	T	-88.8%
8 (Y)	Y	-19.3%	8	Y	540.7%	8	Y	136.8%	8	Y	-19.3%
9 (T)	T	-44.6%	9	T	6527.3%	9	T	-245.8%	9	T	-17.6%

(2) The results obtained by modified iteration process converge to the assigned damage extent if complete DOFs of mode shapes are utilized which demonstrates the superiority to the original iteration process.

(3) The target ratio is proposed to inspect the results of damage quantification. The advantage of the target ratio is that it is very simple and also effective to evaluate the results.

The application of modified MSEC method to a full scale 3-D real frame structure has been studied. Although the iteration process seems to work well for lower modes in the numerical study of the same test structure, but the iteration process fails to converge in the experimental study due to the modeling error, noise effect and the combination of modal expansion from limited number of DOFs. Nevertheless, the damage quantification of the truly damaged sectional properties still clearly identified even though the stiffness reduction factors of others seem to be contaminated.

ACKNOWLEDGEMENTS

REFERENCES

1. Doebling, S. W., Farrar, C. R., Prime, M. B., and Shevitz, D. W. (1996). "Damage identification and health monitoring of structure and mechanical systems from changes in their vibration characteristics: A literature review." Research Rep. No. LA-13070-MS, ESA-EA, Los Alamos National Laboratory, Los Alamos, N.M.
2. Lim, T. W., and Kashangaki, T. A. L. (1994). ''Structural damage detection of space truss structures using best achievable eigenvectors.'' AIAA J., Vol. 32, no. 5, 1049–1057.
3. Doebling, S. W., Hemez, F. M., Peterson, L. D., and Farhat, C. (1997). "Improved damage location accuracy using strain energy based on mode selection criteria." AIAA J., Vol. 35, No. 4, 693–699.
4. Stubbs, N., and Kim, J. T. (1996). ''Damage localization in structures without base-line modal parameters.'' AIAA J., Vol. 34, No. 8, 1644–1649.
5. Shi, Z. Y., Law, S. S., and Zhang, L. M. (1998). ''Structural damage localization from modal strain energy change.'' J. Sound Vib., Vol. 218, No. 5, 825–844.
6. Shi, Z. Y., Law, S. S., and Zhang, L. M. (2000). ''Structural damage detection from elemental modal strain energy change.'' J. Eng. Mech., Vol. 126, No. 12, 1216–1223.
7. Shi, Z. Y., Law, S. S., and Zhang, L. M. (2002). ''Improved Damage Quantification from Elemental Modal Strain Energy Change.'' J. Eng. Mech., Vol. 128, No. 5, 521–529.
8. Hsu, T. Y., Loh, C. H. (2006), "Discussion on Damage Cetection of A 3-D Frame Structure Using Modal Strain Energy Chenge." IMAC XXIV, St. Louis, Missouri, USA, 2006.
9. Thomas R. S., Charles J. C. Joanne L. W. Howard M. A. (1988). ''Comparison of Several Methods for Calculating Vibration Mode Shape Derivatives.'' *AIAA J., Vol. 26, no. 12, 1506–1511.*
10. Kidder, R., "Reduction of Structural Frequency Equations", AIAA Journal, Vol. 11, No. 6, 1973.
11. Ricles, J. M., and Kosmatks, J. B. (1992). ''Damage detection in elastic structures using vibratory residual forces and weighted sensitivity.''AIAA J., 30~9!, 2310–2316.