Statistical Model-based Damage Localization: Implementation with the Scilab Toolbox COSMAD

WENSONG ZHOU, LAURENT MEVEL, MICHÈLE BASSEVILLE, MAURICE GOURSAT, HOUSSEIN NASSER

ABSTRACT:

A free modal analysis and diagnosis toolbox has been designed and successfully applied by some of the authors. It is an identification, detection and localization Scilab toolbox for in-operation situation without any measured or controlled input. The damage localization module and function developed recently within this toolbox is presented in this paper. This damage localization method is based on a residual associated with output-only subspace-based modal identification and global or focused χ^2-tests built on that residual. Moreover, the proposed damage localization approach is also available for a large structure, in such a case, a statistical substructuring method is employed and also implemented with this toolbox. The detailed operations for generating statistical substructuring are also presented. A simulated bridge deck example is reported for explaining the function of toolbox.

INTRODUCTION

The interest in the ability to detect and localize damage of important structures at the earliest possible stage is pervasive throughout the aerospace, mechanical and civil engineering communities. One common approach is to employ vibration characteristics of a structure to predict the damage locations and to estimate the amount of damage, which have been proven useful for health monitoring of mechanical systems [1-3]. It would be very convenient that to integrate such an algorithm into a toolbox, which is expected to provide effective, reliable, accurate and fast on-line assessment for a structure.

A useful free modal analysis and diagnosis Scilab toolbox has been designed and successfully applied by some of the authors [4-9]. This toolbox is developed based on

Wensong Zhou, IRISA/INRIA, zhouwensong@gmail.com;
Laurent Mevel, IRISA/INRIA, lmevel@irisa.fr;
Michèle Basseville, IRISA/CNRS, basseville@irisa.fr;
Maurice Goursat, INRIA, maurice.goursat@inria.fr;
Houssein Nasser, IRISA/Rennes 1 University, hnasser@irisa.fr.

stochastic subspace identification method in time domain. It is identification, damage detection and localization toolbox for in-operation situation without any measured or controlled input. It covers modal identification with visual inspection of the results via a GUI or fully automated modal identification and monitoring. This toolbox can provide the high-quality analysis results, which usually are obtained from some commercial software.

Recently, a damage detection and localization method, based on a residual associated with output-only subspace-based modal identification and global or focused χ^2-tests built on that residual, has been proposed and successfully experimented by some of the authors [10]. Currently, it has been integrated into the COSMAD. Thus, at the first section of this paper, we briefly recall the main theoretical results and the foundations of this damage localization method, and investigate further the damage localization steps. Then, with a simulated bridge deck example, we present the main functions of the toolbox illustrated them with some dialog windows and results.

SUBSPACE BASED MODAL DIAGNOSIS

Modeling and Parameterizations

The structure's behavior is assumed to be described by a stationary linear system:

$$M\ddot{Z} + C\dot{Z} + KZ = v,\ Y = LZ \tag{1}$$

where *M*, *C*, *K* are the mass, damping and stiffness matrices respectively, Z collects the displacements of the degrees of freedom; measurements are collected in Y, and matrix L indicates where the sensors are located. The modes μ, and the modeshapes ψ_μ, are solutions of:

$$\det(\mu^2 M + \mu C + K) = 0,\ (\mu^2 M + \mu C + K)\phi_\mu = 0,\ \ \psi_\mu = L\phi_\mu \tag{2}$$

Sampling model (1) at rate $1/\tau$ yields the discrete time model:

$$\begin{cases} X_{k+1} = FX_k + V_{k+1} \\ Y_k = HX_k \end{cases} \tag{3}$$

The modal parameters defined in (2) are equivalently found from the eigenstructure ($\lambda,\ \varphi_\lambda$) of the state transition matrix *F*:

$$e^{\tau\mu} = \lambda,\ L\Phi_\mu = \phi_\lambda = H\varphi_\lambda \tag{4}$$

Eigenvectors are real if proportional damping is assumed, that is $C = \alpha M + \beta K$. The λ's and ϕ_λ's are pairwise complex conjugate. The collection of modes ($\lambda,\ \phi_\lambda$) forms a canonical parameterization of the pole part of the system in (3). From now on, the collection of modes is also considered as the system parameter θ:

$$\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\Phi \end{pmatrix} \tag{4}$$

where Λ is the vector whose elements are the eigenvalues λ, Φ is the matrix whose columns are the modeshapes φ_λ's, and **vec** is the column stacking operator.

Damage Detection and Localization

In the proposed method, the damage detection is stated as the problem of detecting changes in the canonical parameter vector θ, defined in (4). A residual, which is tightly associated with a relevant parameter estimation method, is defined in this method

[11,12]:

$$\zeta_n(\theta_0) \triangleq \sqrt{n}\,\mathrm{vec}(S^T(\theta_0)\hat{\mathcal{H}}_{p+1,q}) \quad (5)$$

where n is the number of the measured data set, $\hat{\mathcal{H}}$ is the estimation of Hankel matrix built based on the covariance of measured data, matrix S can be obtained using a SVD of the Hankel matrix. For the testing if $\theta = \theta_0$ holds true requires the knowledge of the distribution of $\zeta_n(\theta_0)$. But this distribution is generally unknown, so one manner to circumvent this difficulty is to use the statistical local approach. Specifically, a χ^2–test is employed to decide residual ζ_n is significantly different from zero or not, which should be compared to a threshold:

$$\chi_n^2 \triangleq \zeta_n^T \hat{\Sigma}^{-1} \hat{\mathcal{J}} (\hat{\mathcal{J}}^T \hat{\Sigma}^{-1} \hat{\mathcal{J}})^{-1} \hat{\mathcal{J}}^T \hat{\Sigma}^{-1} \zeta_n \quad (6)$$

where $\hat{\mathcal{J}}$ is the consistent estimate of $\mathcal{J}(\theta_0)$, that is the sensitivities of the residual w.r.t. the monitored parameters; $\hat{\Sigma}$ is the consistent estimate of $\Sigma(\theta_0)$, that is the asymptotical residual covariance.

Damage localization normally is stated as to determine which part of the structure has been changed. For investigating the change of structural parameters, the idea is to express $\mathcal{J}(\theta_0)$ with the structural parameters to be monitored. The mean value of residual ζ_n under the hypothesis of a small deviation $\delta\theta$ in the system parameter from a reference value θ_0 is:

$$E_1(\zeta_n) = \mathcal{J}(\theta_0)\delta\theta \quad (7)$$

Under the assumption of small deviation again, the following relation holds:

$$\delta\theta \approx \mathcal{J}_{\theta\Psi}\delta\Psi \quad (8)$$

where Ψ is the vector of structural parameters to be monitored, and $\mathcal{J}_{\theta\Psi}$ is the Jacobian matrix containing the sensitivities of the mode and mode-shapes w.r.t. those structural parameters. Plugging (8) into (7), the following equation can be obtained:

$$E_1(\zeta_n) = \mathcal{J}(\Psi)\delta\Psi \quad (9)$$

where

$$\mathcal{J}(\Psi) \triangleq \mathcal{J}(\theta_0)\mathcal{J}_{\theta\Psi} \quad (10)$$

For computing the residual sensitivity w.r.t. structural changes given in (10), the Jacobian $\mathcal{J}_{\theta\Psi}$ in (8) need to be computed firstly. For this purpose, we need to transform the discrete modes into the continuous ones, to convert the continuous modes into frequencies and damping coefficients, to match the identified modes with the analytical ones and so on. Altogether, the sensitivities $\mathcal{J}(\Psi)$ defined in (10) writes:

$$\mathcal{J}(\Psi) = \mathcal{J}(\theta_0)\mathcal{J}_{\Phi_i^{(d)}\Phi_i} I_{\Phi_i v_i} \mathcal{J}_{v_i v_a} I_{v_a \Phi_a} \mathcal{J}_{\Phi_a \Psi} \quad (11)$$

where $\mathcal{J}_{\Phi_i^{(d)}\Phi_i}$ is the Jacobian of the transformation of the discrete mode into the continuous ones, $I_{\Phi v}$ is the Jacobian of the conversion of the continuous modes into frequencies and damping coefficients, and $I_{\Phi v}$ is the Jacobian of the inverse conversion, $\mathrm{J}_{v_i v_a}$ corresponds to the manual matching between the identified modes and the analytical ones, $\mathrm{J}_{\Phi_a \Psi}$ is the sensitivities of analytical modes to changes in structural parameters.

Generally, the dimension of the physical parameter space is much higher than that of the modal parameter space, there is a model reduction problem therein. Thus, there are many more columns than lines in matrix $\mathrm{J}(\Psi)$ in (16). Moreover, using a small number of sensors, it is not reasonable to expect the discrimination of all possible structural causes of a given deviation detected by the global damage detection test. To circumvent this difficulty, the idea is to cluster the columns of $\mathrm{J}(\Psi)$ in (16).

In order to make the aggregation operation coherent with the χ^2 decision stage, the metric chosen for performing the clustering is the metric of the χ^2-test. More precisely, let the j-th change direction be defined as the vector:

$$\mathrm{J}_j = \hat{\Sigma}_n^{-T/2} \mathrm{J}(\theta_0) \mathrm{J}_{\Phi_i^{(d)} \Phi_i} I_{\Phi_i v_i} \mathrm{J}_{v_i v_a} I_{v_a \Phi_a} \mathrm{J}_{\Phi_a \Psi}(j) \tag{17}$$

Where $\hat{\Sigma}_n^{-T/2}$ comes from the decomposition $\hat{\Sigma}_n^{-1} = \hat{\Sigma}_n^{-1/2} \hat{\Sigma}_n^{-T/2}$ (always possible since $\hat{\Sigma}_n$ is strictly definite positive) and $\mathrm{J}_{\Phi_a \Psi}(j)$ is the j-th column of $\mathrm{J}_{\Phi_a \Psi}$. In order to cluster these directions into macrofailures, the norm and scalar product of the J_i's are defined:

$$\left\| \mathrm{J}_j \right\|^2 = \mathrm{J}_j^T \mathrm{J}_j, \quad d_{ij} = \frac{\mathrm{J}_j^T \mathrm{J}_j}{\left\| \mathrm{J}_j \right\| \left\| \mathrm{J}_j \right\|} \tag{18}$$

Since we are interested in change directions rather than change magnitudes, the change vectors to be clustered are normalized within this metric. Therefore, the aggregation process should work on the unit sphere, and a classification method able to handle this geometry is needed. For this reason, a vector quantization method of common use in speech processing has been chosen [13]. This method performs a hierarchical classification, while controlling the variability within the classes. For each class, a barycentre C_j is computed. This aggregation mechanism can thus be thought of as a statistical substructuring. Then the χ^2-test writes:

$$\chi_n^2(j) = \tilde{\zeta}_n^T \frac{C_j C_j^T}{\left\| C_j \right\|^2} \tilde{\zeta}_n, \quad \text{where } \tilde{\zeta}_n \triangleq \hat{\Sigma}_n^{-T/2} \zeta_n \tag{19}$$

Assume that $\chi_n^2(j)$ exceeds a given threshold. Then, all the structural elements within the class corresponding to the barycentre C_j are possible causes of the detected damage.

IMPLEMENTATION WITH COSMAD TOOLBOX

The steps of localization are summarized as following:

1) Edit and prepare the data files, structural geometry files and other required files;
2) Run the modal identification with subspace method, obtain the nominal model θ_0;
3) Data preprocessing, namely compute the estimates of the sensitivity and residual covariance matrices $\mathrm{J}(\theta_0)$, $\Sigma(\theta_0)$;

4) Apply the χ^2-tests on both the reference data and the possible damaged data, and evaluate the possible structure change;
5) Compute the sensitivities of analytical modes to structural changes $\mathcal{J}_{\theta\Psi}$, or specially import from an existed external data files;
6) Match between the identified modes and the computed ones;
7) Make Jacobian fusion between $\mathcal{J}(\theta_0)$ and $\mathcal{J}_{\theta\Psi}$;
8) Cluster the fused Jacobian for a large-scale structure to different classes with a statistical substructuring method;
9) Apply the χ^2-tests on each class and compare with a given threshold to assess which class affected by the damage.

Step 1-4 are also the procedure of identification and detection, their detailed description has already been given clearly in ref. [4-7]. Thus, in the following with a simulated bridge deck example, we focus on describing the last 5 steps mainly since we are concerned only about the implementation of damage localization.

Working Example

The working example is a bridge deck (3m in height, 6.6 to 10 m in width)[10], shown in Figure 1. A finite element model, in which the 100 m span is modeled using 9600 volume elements and 13668 nodes, has been developed. The damaged elements, whose location is 16.5 m from the end, are modeled as a reduction of the material modulus by up to 30%. Exactly, the damaged area consists of two sections, each of which includes 48 elements. But only 44 elements among them are simulated as damaged, see Figure 2, so there are total 88 damaged elements within the deck. The output are simulated under white noise excitations using 21 sensors, which are placed evenly along the bridge, also see Figure 1.

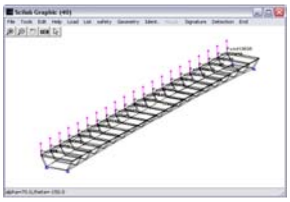


Figure 1: The example deck and sensors plotted with COSMAD

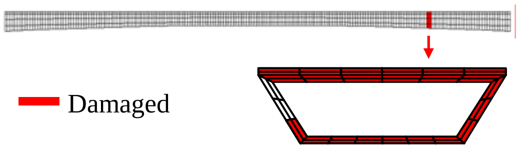


Figure 2: The damage elements in the deck

Identification and Data preprocessing

Before localization, we need perform modal identification and data preprocessing with COSMAD. The modal identification combines two types of activities: 1) computational steps of the subspace algorithms, and 2) exploitation of the results. Figure 3 is the signal processing and selection window. Figure 4 shows the dialog windows for the case of a detailed inspection of the results of a global automatic identification procedure.

Matching Modes

The "Localisation" command and submenus are shown in Figure 5. It is enabled after the preprocessing step. Starting with the "New" command we can carry out the localization step by step following the menu. Before matching modes, two data files are need: the file containing the modal sensitivities data and the finite element signature file containing the analytical information. For this example, the modal sensitivities are

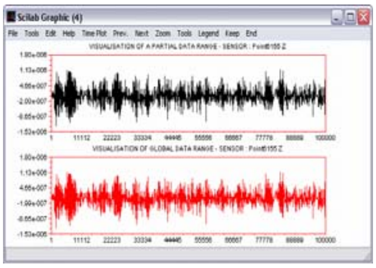


Figure 3: Basic signal processing and selection

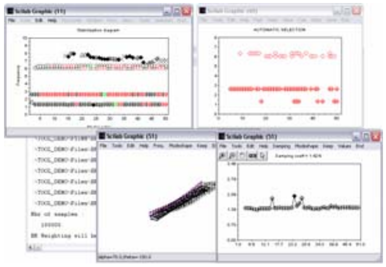


Figure 4: Results of the automatic selection

computed with SDTOOLS in advance. Then after loading two essential files, we can match the modes of the identified signature with the modes of the FEM. This step is manual: we have to choose those corresponding modes by click. The MAC (*Modal Assurance Criterion*) plots are used to determine the matching pair of modes. Figure 6(a)-(d) show the good matching between two signatures for the first 4 modes, (e)-(g) show the frequency and MAC values for all pairs of modes and displays the same information in the form of histogram. Sometimes some identified modes can not be found among the analytical modes, in this case the modal signature need to be updated. The estimations of the sensitivity and residual covariance matrices also need compute again. This step will be finished with the help of prompt messages. After matching modes, all matching information will be saved into an external file for other steps using.

Jacobian Fusion

According to the matching results, the sensitivities matrix of the modal parameter to the residual vector are combined with the corresponding sensitivities of analytical modes to structural changes, namely to implement Eq.(11). This step is called Jacobian Fusion. It is performed background with message prompt by running the command "Jacobian Fusion" in the "Localization" menu.

Clustering

For this example, the number of structural parameters (9628) is much larger than the modes number (4). In this case, as stated above, we need cluster the Jacobian $\mathrm{J}\ (\Psi)$ matrix into different classes. In COSMAD, the "Clustering" command in "Localisation" menu will be available after "Jacobian Fusion" operation finished. When running the "Clustering", user need select two thresholds/parameters 1) Covariance cluster and 2) Max number of class for determining the magnitude and number of classes: normally the smaller covariance cluster will lead to more number of classes; specially setting it zero will lead to no clustering. Thus, when "Clustering" finished, we obtain the barycentre C_j for each class.

Run Localization on the Other Data Set

Using a set of fresh data to assess if the structure is damaged or not, we need compute the new residual vectors and apply the χ^2-tests on each class. The user need list this data record with "List" menu firstly, and then run localization on this set of data.

We can now display the results with display windows shown as Figure 7. We may choose "Test" for displaying the corresponding test value for each class and determining which class is damaged as Figure 8. For the example in this paper, it is easy to determine the 73rd class is damaged. Figure 9 shows the location of the 73rd class in the bridge deck. We can also see the other class as Figure 10.

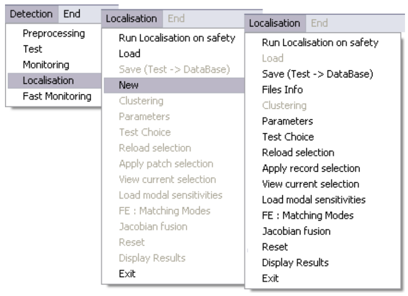
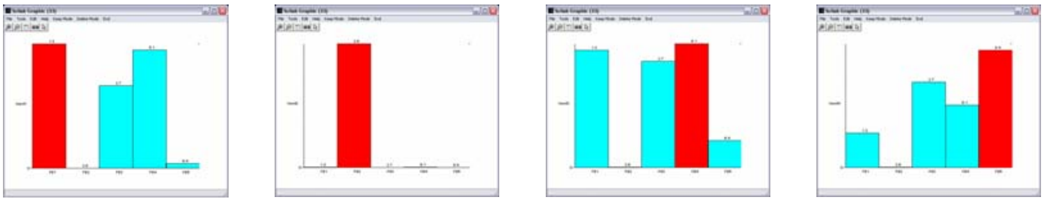
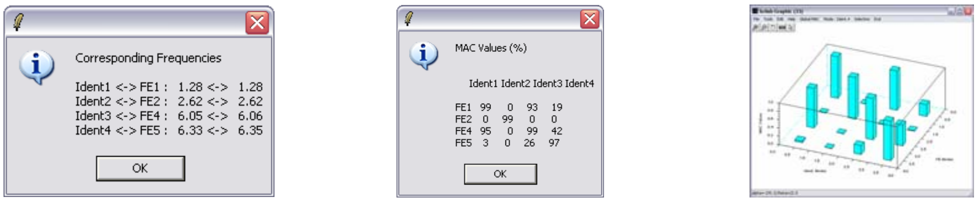


Figure 5: The localization menu and functions



(a) (b) (c) (d)



(e) (f) (g)

Figure 6: Matching of modes: results and plots

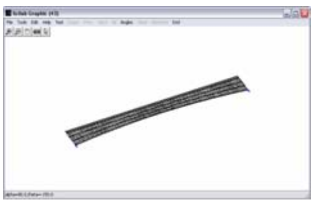


Figure 7 Localization: results display

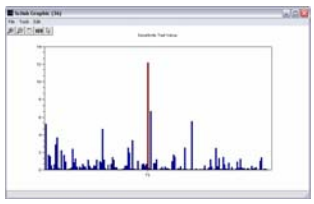


Figure 8 Localization: test value

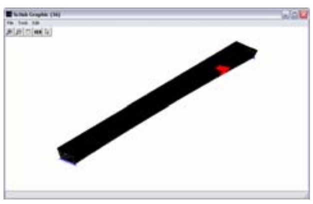


Figure 9 Localization: the damaged class

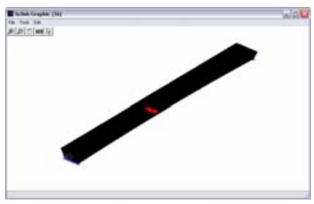


Figure 10 Localization: the other class

CONCLUSIONS

In this paper, we present the new localization module and function of the Scilab toolbox COSMAD by showing interface windows and some results of a new example on a simulated bridge deck. After describing the proposed damage localization method, which based on both a subspace residual and on a statistical analysis of aggregated sensitivities of the residual to the damages, the detailed damage localization steps are described in this paper. With a large structure example, the detailed operations for

generating statistical substructuring are also presented.
The COSMAD toolbox is available at www.irisa.fr/sigma2/constructif/modal.htm. More detailed function, instruction and demo of this toolbox can be found in this website. Scilab scientific package is available at www.scilab.org.

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