Damage Identification Based on a Local Physical Model for Small Clusters of Wireless Sensors

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ABSTRACT

Structural assessment is becoming increasingly important in civil engineering. We present a decentralized method for local damage identification using a dense array of wireless sensors. The method is based on second-order ARX models as an approximation of a local model of the structure.

The method is applied to numerical data from a building finite element model similar to that of the ASCE Benchmark. It is shown that damage on individual structural elements can be identified and located.

INTRODUCTION

Recent developments in the wireless sensing area have opened many possibilities for local damage detection in civil structures: when they become low-cost, it will be possible to put numerous sensors on a structure, and identify local damage. In this regard, it is important to embed algorithms on sensors to ensure network scalability: transmitting only the results of local computations reduces data exchange, so that communications are not a limiting factor.

In 2001, Sohn and Farrar [1] presented a novel procedure to identify damage from the acceleration time histories of one sensor, using a two-stage AR-ARX model and a damage-sensitive feature linked to the prediction error of that model. Lei et al. [2] proposed a modification to this method, and Lynch et al. [3] embedded the method on their wireless platform, thereby showing its energy efficiency.

In a previous paper [4], we presented two methods using multivariate models and based on the method developed by Sohn and Farrar [1], and examined their effectiveness using the ASCE Phase I Analytical Benchmark [5]. In this paper, we further investigate the second damage identification method.

It requires using data from a few closely-spaced sensors. It is believed that more accurate information about the structural dynamics can be obtained by using several channels in the analysis, rather than only one. This method is based on the local physical model of one structural node: using finite differences, the second-order dynamics of that node are rewritten as a multivariate ARX(2,2) model involving the accelerations of all neighboring

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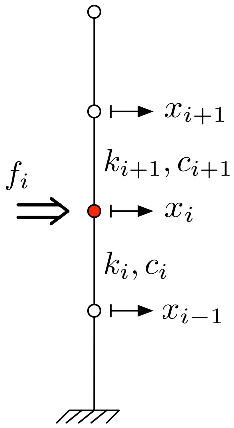

Figure 1: Lumped-mass unidimensional shear model

nodes.

The parameters of this ARX model are identified for a pool of reference acceleration time histories when the structure is healthy. When a new time history is recorded, it is compared to the reference model that was built under closest environmental conditions to the current ones. If the residual error of the predicted output is greater than that of the reference time history, then structural members to which the node belong are identified as damaged. This analysis may be performed at various locations of the structure, providing evaluation of existence and location of damage.

Numerical verification of this approach is performed using data from a finite element model of a 4-story building. The model has the same structural properties as that of the ASCE Benchmark [5].

BACKGROUND

Local Physical Model

Since details of the model have already been discussed in Monroig and Fujino [4], only a brief presentation follows.

Consider a simple lumped mass unidimensional shear model as presented in Figure 1. The equation of dynamics for the node i is

$$m_i\ddot{x}_i + c_i\left(\dot{x}_i - \dot{x}_{i-1}\right) + c_{i+1}\left(\dot{x}_i - \dot{x}_{i+1}\right) + k_i\left(x_i - x_{i-1}\right) + k_{i+1}\left(x_i - x_{i+1}\right) = f_i \tag{1}$$

After differentiating twice and approximating the first and second derivatives of the accelerations by finite differences

$$\frac{da_i}{dt}(t) = \frac{a_i(t) - a_i(t-T)}{T} \quad , \quad \frac{d^2a_i}{dt^2}(t) = \frac{a_i(t+T) - 2a_i(t) + a_i(t-T)}{T^2} \tag{2}$$

where T is the sampling interval, Eq (1) is reduced to an ARX(2,2) model

$$y(t) + A_1y(t-1) + A_2y(t-2) = B_1u(t-1) + B_2u(t-2) + e(t) \tag{3}$$

where $y(t) = a_i(t)$ is the system output, $u(t) = \begin{bmatrix} a_{i-1}(t) \\ a_{i+1}(t) \end{bmatrix}$ is the system input and $e(t)$ incorporates both the second derivative of the external force at node i, and the model and measurement errors. Here, the notation T has been replaced by 1 for convenience of notations.

Damage Identification

Using the model, a damage identification procedure was developed similar to the one presented by Sohn and Farrar [1]. The procedure is divided into two steps: normalization and feature extraction.

NORMALIZATION PROCEDURE

It is assumed that for each cluster, we have collected acceleration response time histories of the structure in its healthy state, into a *reference database*. In the cluster, the channels are separated into input and output channels as described previously, and the time histories as a whole is written as $x(t) = (x_{input}(t), x_{output}(t)) = (x_I(t), x_O(t))$.

For each of these signals, the mean is removed[1], and then the coefficients of an ARX(2,2) model are identified and stored in the database.

Having measured a new signal $y(t) = (y_I(t), y_O(t))$, it is also processed in the same way, and its ARX(2,2) model also identified. The coefficients are then compared to the database coefficients in order to select a reference signal $x_0(t)$ that is *closest* to $y(t)$. Although we tried to use the Euclidian distance between the coefficients, some of the time histories could not be properly matched to the correct input category. Therefore, in this paper, we temporarily assigned to each time history a reference time history with the same input case (I, II or III).

FEATURE EXTRACTION

Having selected the closest reference signal $x_0(t)$, the prediction error of the associated reference model is computed for the new signal $y(t)$:

$$e_y(t) = y_O(t) + A_1^{x_0} y_O(t-1) + A_2^{x_0} y_O(t-2) - B_1^{x_0} y_I(t-1) - B_2^{x_0} y_I(t-2) \quad , \qquad (4)$$

The prediction error $e_y(t)$, as well as the output time history $y_O(t)$, are multi-channel signals. Since we did not normalize the measured accelerations, we cannot directly compare this prediction error $e_y(t)$ to the reference one, $e_{x_0}(t)$, since the excitation level might be different. Therefore, for each output channel we first normalize the prediction error with respect to the measured acceleration:

$$\hat{e}_{y,i}(t) = e_{y,i}(t) / \sigma(y_{O,i}(t)) \quad \text{for all channels } i \qquad (5)$$

If the structure is damaged, the standard deviation of the now normalized prediction error $\hat{e}_{y,i}(t)$ should increase compared to that of $\hat{e}_{x_0,i}(t)$. The ratio of the standard deviations is thus selected as damage sensitive features, and damage detection is be reduced to a statistical test. The null hypothesis is $H_0 : \sigma^2(\hat{e}_{y,i}(t)) = \sigma^2(\hat{e}_{x_0,i}(t))$, the alternative being $H_1 : \sigma^2(\hat{e}_{y,i}(t)) > \sigma^2(\hat{e}_{x_0,i}(t))$. The generalized F-test for the ratio of variances described by Sohn and Farrar [1] was used, that doesn't assume normality of the prediction errors.

[1] But unlike in Monroig and Fujino [4], the signals are not normalized to have a unit variance in each channel.

NUMERICAL SIMULATION

Model

A finite element model of a steel frame building was developed using the general purpose finite element software *Abaqus*. It is similar to the model that appears in the ASCE Phase I Analytical Benchmark [5], in the sense that it models the same structure, but has approximately 12600 degrees-of-freedom.

The model is a 4-story 2-bay by 2-bay steel frame structure, with four concrete slabs on each floor. The geometrical properties of the models, as well as the mechanical properties of the beams, columns and braces were kept identical to those of the ASCE Benchmark model [5].

The first difference with the ASCE Benchmark model is that the concrete slabs are modeled as linear elastic shell elements, with properties shown in Table 1. The thickness was estimated from the mass of each slab.

The second difference is that the model has been refined, with 10 beam elements for each beam, but only one for the cross-bracings, which are modeled as truss elements. The objective is to be able to observe the vibrations of each element, and hence to detect damage of individual elements.

Finally, damping was modeled as modal damping, increasing linearly with respect to frequency from 0.02% of critical damping at 10 Hz to 0.1% at 200 Hz, and constant after 200 Hz.

In order to take into account vibrations of individual elements, the data was generated with a sampling rate of 5000 Hz, and modes up to 5000 Hz were taken into account.

Input Excitations

Three different input cases were defined, all involving excitations modeled as independent unfiltered Gaussian white noise, generated at a sampling rate of 5000 Hz. The locations and amplitudes of the excitations are represented in Figure 2.

Input case (I) has one random excitation at the middle node of the fourth floor on the North side, directed along the y axis, and modeled as Gaussian white noise with zero mean and 10^6 N standard deviation.[2]

Input case (II) has one random excitation at the middle node of the second, third and fourth floors on the North side, all directed along the y axis. The random excitations are modeled as Gaussian white noise with mean 10^6 N and standard deviation 10^5 N for the

[2] It should be noted that the simulated model is linear, so that only the relative values of the excitation levels are important.

Table 1: Properties of the four slabs of each floor of the FEM model

Property	1^{st} floor	2^{nd},3^{rd} floors	4^{th} floor
Young's modulus E (Pa)	3.0×10^{10}	3.0×10^{10}	3.0×10^{10}
Poisson ratio ν	0.15	0.15	0.15
Mass per unit volume ρ (kg/m^3)	2,450	2,450	2,450
weight (kg)	800	600	400
thickness (m)	0.209	0.157	0.104

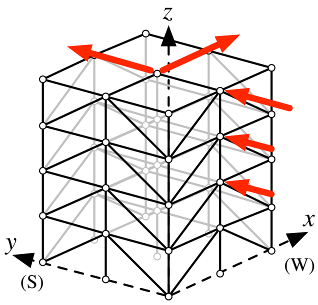

Figure 2: Input excitation locations

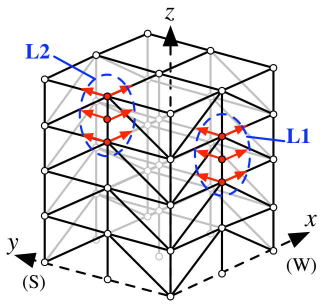

Figure 3: Local Clusters

second and third floors, and with mean 10^7 N and standard deviation 10^6 N for the fourth floor.

Input case (III) is the same as (II) but without the mean component, and with an additional random excitation oriented diagonally at the center node of the fourth floor, modeling a shaker (Figure 2 shows the components of the load). The random excitations are modeled as Gaussian white noise with standard deviation 10^5 N for the second and third floors, 10^6 N for the fourth floor, and 4×10^6 N for the shaker.

Damage Cases

We defined six damage cases similar but different to the ones in the ASCE Benchmark. In all cases, damage is simulated by reducing the stiffness and mass of the elements.[3] Damage cases 3 to 6 are subdivided into several subcases where the damage locations are the same, but the reduction coefficients vary.

Damage cases are presented in Figure 4, and are as follows:

(1) 90 % reduction in all braces of the first floor;

(2) 90 % reduction in all braces of the third floor;

(3) 90 %, 75 %, 50 % and 25 % reduction in one brace of the third floor (west brace on north face);

(4) 90 %, 75 %, 50 % and 25 % reduction in one column of the third floor (middle column on north face);

(5) 90 %, 75 %, 50 % and 25 % reduction in one column of the third floor (middle column on north face) and in one column of the fourth floor (middle column on east face);

(6) 75 %, 50 % and 25 % reduction in one brace of the second floor (south brace on east face); 75 %, 50 % and 25 % reduction in one column of the fourth floor (middle column on east face); 50 %, 75 % and 50 % reduction in one brace of the third floor (west brace on north face);

Sensors

Acceleration time histories were produced for all excitation cases, for the healthy structure and for each damage case. Accelerometers were grouped into clusters of three sensors each, as shown in Figure 3. Only y-axis accelerations have been analyzed in this paper.

[3]For braces, the cross-sectional area is reduced. For beams, the cross-sectional area, the moments of inertia and the torsion constant are all reduced.

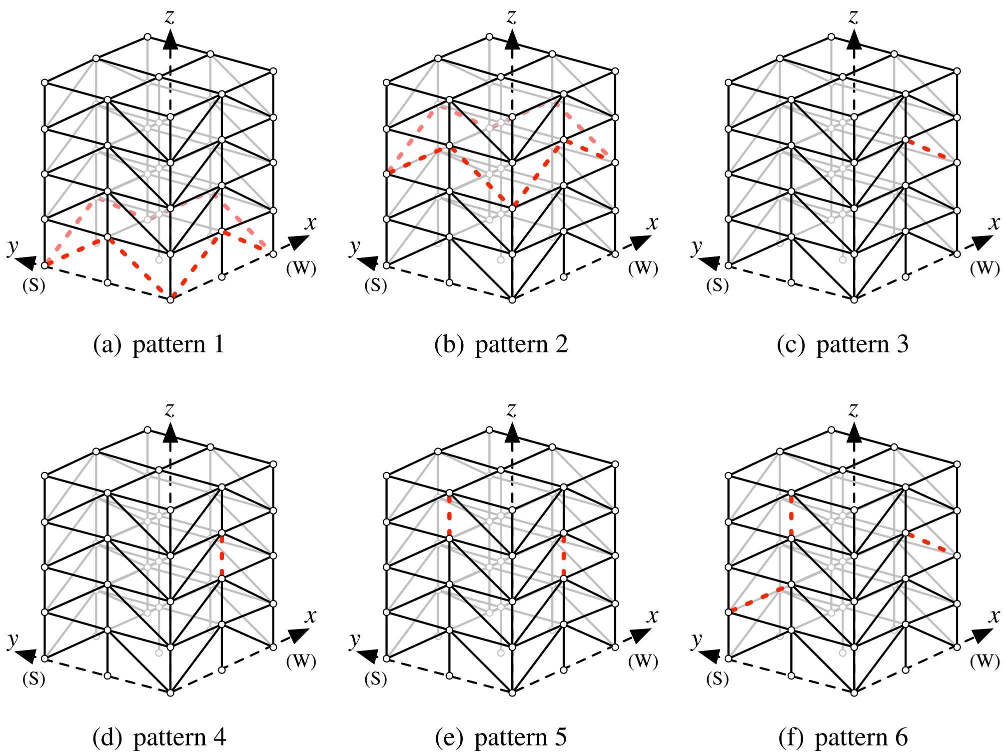

Figure 4: Damage cases 1 to 6

The *local clusters* in Figure 3 are groups of sensors belonging to the same structural element, defined to detect damage in individual elements. The acceleration time histories for the local clusters are not downsampled.

20 time histories are generated for the healthy structure for each input case to build the database, and 10 time histories are generated for each input case for each damage pattern. In other words, there are 60 "healthy" time histories, and 30 time histories for each damage pattern. The time histories are five seconds long, and therefore have 25000 data points each.

For all time histories, Gaussian white noise is added to all acceleration channels. The noise level is set to be the same for all channels of one cluster, with a standard deviation of 1 % of the highest channel standard deviation.

RESULTS

The damage identification results for both local clusters are presented in Table 2. For each damage case from 1 to 6, the stiffness and mass reduction percentage is written on the left side for reference. On the right side, are shown the number of times the null hypothesis was rejected (the structure was identified as damaged) out of the 30 time histories for each damage case. The test was conducted with a confidence interval of 97.5%.

Table 2: Damage identification results. For each damage case from 1 to 6, the stiffness and mass reduction percentage is written on the left side for reference. On the right side, are shown the number of times the null hypothesis was rejected (the structure was identified as damaged) out of the 30 time histories for each damage case. The test was conducted with a confidence interval of 97.5%.

damage case	stiffness & mass reduction (%)		damage identification	
	L1	L2	L1	L2
1	0	0	3/30	0/30
2	0	0	8/30	0/30
3a	0	0	2/30	0/30
3b	0	0	1/30	0/30
3c	0	0	2/30	0/30
3d	0	0	0/30	0/30
4a	90	0	30/30	0/30
4b	75	0	30/30	1/30
4c	50	0	18/30	0/30
4d	25	0	12/30	0/30
5a	90	90	30/30	30/30
5b	75	75	30/30	30/30
5c	50	50	20/30	30/30
5d	25	25	12/30	29/30
6a	0	75	3/30	30/30
6b	0	50	3/30	30/30
6c	0	25	1/30	29/30

For the cluster L2, all or almost all time histories were identified as damaged for damage cases 5a, 5b, 5c, 5d, 6a, 6b and 6c. Also, there were none or very few false positive damage identification for the other damage cases, even for damage cases 1 and 2 which are globally damaging the structure.

For the cluster L1, all time histories were identified as damaged for damage cases 4a, 4b, 5a and 5b. However, not all time histories were identified as damaged for cases 4c, 4d, 5c and 5d, which are less severe stiffness and mass reduction. False-positive damage identification results are observed for almost all other damage patterns.

It is concluded that all damage patterns were properly identified for cluster L2. Also, for cluster L1, damage patterns where the stiffness and mass reduction in the corresponding column is 75% or greater were properly identified, but there remains some false-positive and false-negative identification results for the other damage patterns.

SUMMARY

In this paper, a decentralized method for local damage identification was presented and verified on numerical data. The method is based on second-order ARX models as an approximation of a local model of the structure.

The method was applied to numerical data from a building finite element model similar to that of the ASCE Benchmark. It is shown that damage on individual structural elements can be identified and localized.

Although there are false-positive and false-negative results, the method shows promise as a simple local damage identification algorithm. Further research is underway to improve it.

REFERENCES

1. Hoon Sohn and Charles R. Farrar. Damage diagnosis using time series analysis of vibration signals. *Smart Materials and Structures*, 10(3):446–451, June 2001.
2. Y. Lei, A. S. Kiremidjian, K. K. Nair, J. P. Lynch, K. H. Law, T. W. Kenny, Ed Carryer, and A. Kottapalli. Statistical damage detection using time series analysis on a structural health monitoring benchmark problem. In *Proceedings of the 9th International Conference on Applications of Statistics and Probability in Civil Engineering*, San Francisco, CA, USA, July 6-9 2003.
3. Jerome P. Lynch, Arvind Sundararajan, Kincho H. Law, Anne S. Kiremidjian, and Ed Carryer. Design of wireless sensor units with embedded statistical time-series damage detection algorithms for structural health monitoring. In *Caltrans/UCSD Workshop on Structural Health Monitoring and Instrumentation and Diagnostics of Bridge Infrastructure*, San Diego, CA, USA, March 7-8 2003.
4. Evan Monroig and Yozo Fujino. Multivariate autoregressive models for local damage detection using small clusters of wireless sensors. In *Proceedings of the Third European Workshop on Structural Health Monitoring*, Granada, Spain, July 5-6 2006.
5. E. A. Johnson, H. F. Lam, L. S. Katafygiotis, and J. L. Beck. Phase I IASC-ASCE structural health monitoring benchmark problem using simulated data. *Journal of Engineering Mechanics*, 130(1):3–15, January 2004.
6. Lennart Ljung. *System Identification: Theory for the User*. Upper Saddle River, NJ: Prentice Hall PTR, second edition, 1999.