Cepstral Metric for Structural Health Monitoring

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ABSTRACT:

A new damage indicator, denoted by cepstral metric for ARMA models, is proposed for application of damage detection in civil engineering. Two approaches calculating this cepstral metric by using either the poles and zeroes or subspace angles are introduced. The verification simulation indicates that the cepstral metric is qualified for a sensitive damage indicator both on damage location and damage level.

1. INTRODUCTION

Over the past two decades, structural health monitoring (SHM) has been getting strong attention for maintaining proper performance of building structures against natural hazards such as large earthquakes and strong winds [1]. To seek for a sensitive damage indicator that can accurately distinguish a damaged structure from an undamaged one is the focus of most SHM technical literature. Many researchers have proposed the modal parameters, such as modal frequency or modal shape, as the primary damage indicator due to the observation that changes in structural properties, such as mass, damping and stiffness, will cause changes in modal parameters. However, the modal parameters are often less sensitive to local change in structure [2], which in some cases becomes a major limitation for the vibration-based methods which rely on identifying these parameters to locate and assess damage.

Alternatively, a statistical pattern recognition methodology, which utilize linear time series model fitting vibration measurements for the identification of damage in civil structures, was given in [3]. Sohn and Farrar proposed a two-stage linear prediction model, combining Auto-Regressive (AR) model and Auto-Regressive model with exogenous inputs (ARX). The residual error, which is the difference between the actual acceleration measurements and the prediction from the AR-ARX model, is defined as the damage indicator. In their method, the Euclidean metric for measuring difference of AR coefficients is used to determine if there is strong agreement between two time series models.

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In fact, the AR-ARX modeling is similar to a linear approximation method of an Auto-Regressive Moving-Average (ARMA) model. ARMA model is a useful single-input single-output linear time invariant (SISO-LTI) model for the representation of discrete-time signals [1]. It is apparent that ARMA estimation can be used for time series classification. For classification, we need find a metric measuring the distance between ARMA models. Concentrating on the AR case, we might use the metric in terms of AR coefficients mentioned above. This approach would raise some problems, that is, this metric does not have any system-theoretic and mathematical properties. To overcome this difficulty, a new metric for ARMA models in terms of cepstrum has been proposed by Martin [4]. As a complementation, De Cock [5] proposed the subspace angle between ARMA models and related it to the cepstral metric defined by Martin.

In this paper, we attempt to explore a new application of the cepstrum to damage detection in civil engineering by taking the cepstral metric defined by Martin as the damage indicator. We begin with introducing two approaches to calculate the cepstral metric using either the poles and zeroes or subspace angles of ARMA models. The equivalent relation between these two approaches is introduced then. For SHM, whether a structure is damaged or not and how seriously the damage occurred are determined by using this cepstral metric to measure the distance between modeled time series from reference and unknown structures. A numerical five-story shear building is used to examine the performance of this damage indicator expressed by cepstral metric, which is verified to be qualified for a sensitive damage indicator both on damage location and damage level.

2. CEPSTRAL METRIC FOR ARMA MODELS

The cepstrum (natural pronunciation would be kepstrum) was firstly introduced by Bogert, Healy and Tukey, who used it for the detection of echoes. Cepstrum has been applied in a variety of areas including audio processing, speech processing, geophysics, medical imaging, and others [6]. In this section, we will discuss two approaches of calculating the cepstral metric ARMA models proposed by Martin. It will be shown that these two methods have a skillful equivalence.

2.1. Cepstral Metric in terms of Poles and Zeroes

The power cepstrum of a SISO-LTI model with transfer function $H(z)$ is the inverse Fourier transform of the logarithm of its power spectrum $P(z)$ [7],

$$\log P(z) = \log H(z)\overline{H}\left(z^{-1}\right) = \sum_{n\in z} C_n z^{-n} \qquad (1)$$

where C_n are the cepstrum coefficients.

An autoregressive moving-average (ARMA) model, which is a general parametric description of SISO-LTI process, is given by

$$x_n = -\sum_{i=1}^{p} a_i x_{n-i} + \sum_{i=0}^{q} b_i e_{n-i}\ , \qquad (2)$$

where a_i and b_i are the AR and MA coefficients, respectively. p and q are the model orders of the AR and MA processes, respectively, and e_n is a white noise process with zero mean and variance σ^2. The transfer function of a stable, minimum ARMA process has the form in the z domain

$$H(z) = \frac{\sum_{i=0}^{q} b_i z^{-i}}{\sum_{i=0}^{p} a_i z^{-i}} = \frac{\prod_{i=1}^{q}\left(1-\beta_i z^{-1}\right)}{\prod_{i=1}^{p}\left(1-\alpha_i z^{-1}\right)} \qquad (3)$$

Where α_i and β_i are the poles and zeros of the ARMA model. If we compute the logarithm of the power spectrum

$$\log P(z) = -\sum_{i=1}^{p} \log\left|z-\alpha_i\right|^2 + \sum_{i=1}^{q} \log\left|z-\beta_i\right|^2 + \log \sigma^2, \qquad (4)$$

by combining (1) with (4), the power cepstrum can be expressed by the poles and zeroes of the ARMA model as following

$$C_n = \begin{cases} \frac{1}{|n|}\left[\sum_{i=1}^{p} \alpha_i^{|n|} - \sum_{i=1}^{q} \beta_i^{|n|}\right], & n \neq 0, \\ \log \sigma^2, & n = 0, \end{cases} \qquad (5)$$

Martin defined the metric for the set of SISO LTI ARMA models, which is based on the cepstrum [4]. For ARMA models $M^{(1)}$ and $M^{(2)}$ with cepstrum coefficients defined as $C_n^{(1)}$ and $C_n^{(2)}$, the cepstral metric is

$$D\left(M^{(1)}, M^{(2)}\right)^2 = \sum_{n=1}^{\infty} n\left|C_n^{(1)} - C_n^{(2)}\right|^2 \qquad (6)$$

This is a Euclidean metric, which induces the following property on the set of ARMA models:

$$D\left(M^{(1)}M^{(3)}, M^{(2)}M^{(3)}\right) = D\left(M^{(1)}, M^{(2)}\right) \qquad (7)$$

In other words, if two models $M^{(1)}$ and $M^{(2)}$ are passed through the same linear filter with model $M^{(3)}$, their mutual distance is unaltered.

Therefore, if $H_{ARMA}^{(1)}(z) = \frac{b^{(1)}(z)}{a^{(1)}(z)}$ and $H_{ARMA}^{(2)}(z) = \frac{b^{(2)}(z)}{a^{(2)}(z)}$ are the transfer functions of two ARMA models $M^{(1)}$ and $M^{(2)}$, and by putting $M^{(3)}$ with transfer function $H^{(3)}(z) = \frac{1}{b^{(1)}(z)b^{(2)}(z)}$, we may construct two AR models $N^{(1)}$ and $N^{(2)}$ with transfer functions:

$$\begin{cases} H_{AR}^{(1)}(z) = H_{ARMA}^{(1)}(z)H^{(3)}(z) = \frac{1}{a^{(1)}(z)b^{(2)}(z)} \\ H_{AR}^{(2)}(z) = H_{ARMA}^{(2)}(z)H^{(3)}(z) = \frac{1}{a^{(2)}(z)b^{(1)}(z)} \end{cases} \qquad (8)$$

Consequently, we have then

$$D\left(M^{(1)}, M^{(2)}\right) = D\left(N^{(1)}, N^{(2)}\right), \qquad (9)$$

That's to say, it is sufficient to deal with ARMA models by considering AR models only. Thus, for two stable AR models $M^{(1)}, M^{(2)}$ with order $p^{(1)}, p^{(2)}$ and poles $\alpha_i^{(1)}, \alpha_i^{(2)}$, the cepstral metric can be expressed by the poles of AR models as following:

$$D\left(M^{(1)}, M^{(2)}\right)^2 = \log \frac{\prod_{i=1}^{p^{(1)}} \prod_{j=1}^{p^{(2)}} \left(1-\alpha_i^{(1)}\overline{\alpha}_j^{(2)}\right) \prod_{i=1}^{p^{(2)}} \prod_{j=1}^{p^{(1)}} \left(1-\alpha_i^{(2)}\overline{\alpha}_j^{(1)}\right)}{\prod_{i=1}^{p^{(1)}} \prod_{j=1}^{p^{(1)}} \left(1-\alpha_i^{(1)}\overline{\alpha}_j^{(1)}\right) \prod_{i=1}^{p^{(2)}} \prod_{j=1}^{p^{(2)}} \left(1-\alpha_i^{(2)}\overline{\alpha}_j^{(2)}\right)} \qquad (10)$$

2.2. Cepstral Metric in terms of Subspace Angles

A SISO-LTI stable, minimum phase ARMA model can be described in the forward innovation state space form:

$$\begin{cases} x(k+1) = Ax(k) + Ku(k) \\ y(k) = Cx(k) + u(k) \end{cases} \qquad (11)$$

where $y(k)$, the output of the model, is the stochastic process that is being modeled, $e(k)$ is the innovation process of $y(k)$ and $x(k)$ is the state process. The matrix A is called the system matrix, C is the output matrix and K is the Kalman gain. This model can also be denoted by the threesome (A, K, C), the poles of which are the eigenvalues of the system matrix A. We may define the associated infinite observability matrix:

$$\mathcal{O}_\infty = \begin{bmatrix} C & CA & CA^2 & \cdots \end{bmatrix}^T \qquad (12)$$

From the above model (12), we can immediately derive the state space equations of the inverse model:

$$\begin{cases} x(k+1) = (A-KC)x(k) + Ky(k) \\ u(k) = -Cx(k) + y(k) \end{cases} \qquad (13)$$

The zeroes of the model (A, K, C) are consequently equal to the eigenvalues of $(A-KC)$. The infinite observability matrix of the inverse model is denoted by

$$\mathcal{O}_{i\infty} = \begin{bmatrix} -C & -C(A-KC) & -C(A-KC)^2 & \cdots \end{bmatrix}^T \qquad (14)$$

Let $M^{(1)}$ and $M^{(2)}$ be two stable, minimum phase ARMA models of order n. $\mathcal{O}_\infty^{(1)}$ and $\mathcal{O}_\infty^{(2)}$ are the observability matrices of two models, and $\mathcal{O}_{i\infty}^{(1)}$ and $\mathcal{O}_{i\infty}^{(2)}$ are the observability matrices of the inverse models, respectively. De Cork defined the subspace angles between $M^{(1)}$ and $M^{(2)}$ as the principal angles $\theta_i (i = 1, \cdots, 2n)$ between the ranges of $\left(\mathcal{O}_\infty^{(1)} \quad \mathcal{O}_{i\infty}^{(2)}\right)$ and $\left(\mathcal{O}_\infty^{(2)} \quad \mathcal{O}_{i\infty}^{(1)}\right)$ [5].

Note that, the cepstral metric of ARMA models defined by Martin can be related to the subspace angles between ARMA models. For two ARMA models $M^{(1)}$ and $M^{(2)}$ of order n and subspace angles $\theta_i (i = 1, \cdots, 2n)$ between them, the metric in terms of subspace angles is equal to

$$D\left(M^{(1)}, M^{(2)}\right)^2 = \log \prod_{i=1}^{2n} \frac{1}{\cos^2 \theta_i} \qquad (15)$$

Analogously, assume two stable and observable AR models $M^{(1)}$ and $M^{(2)}$ are characterized in state space form by their system matrices $A^{(1)}$ and $A^{(2)}$ and output matrices $C^{(1)}$ and $C^{(2)}$, respectively. The associated infinite observability matrices are $\mathcal{O}_\infty^{(1)}$ and $\mathcal{O}_\infty^{(2)}$, respectively. If we define the subspace angles between $M^{(1)}$ and $M^{(2)}$ as the principal angles $\theta_i\left(i=1,\cdots,2n\right)$ between the ranges of $\mathcal{O}_\infty^{(1)}$ and $\mathcal{O}_\infty^{(2)}$, we may also relate the cepstral metric in terms of poles of AR models (11) to subspace angles between AR models. For two AR models $M^{(1)}$ of order $n^{(1)}$ and $M^{(2)}$ of order $n^{(2)}$ with subspace angles $\theta_i\left(i=1,\cdots,2n\right)$, the metric in terms of subspace angles is equal to

$$D\left(M^{(1)},M^{(2)}\right)^2 = \log\prod_{i=1}^{n}\frac{1}{\cos^2\theta_i}, \qquad (16)$$

where $n=\max\left(n^{(1)},n^{(2)}\right)$

3. CEPSTRAL METRIC FOR DAMAGE DETECTION

In this section, a new damage indicator denoted by the cepstral metric described above is proposed to identify the structural damage for SHM. This methodology is applied to a numerical model of five-story shear building, as shown in Figure 1, for evaluating the performance of this damage indicator.

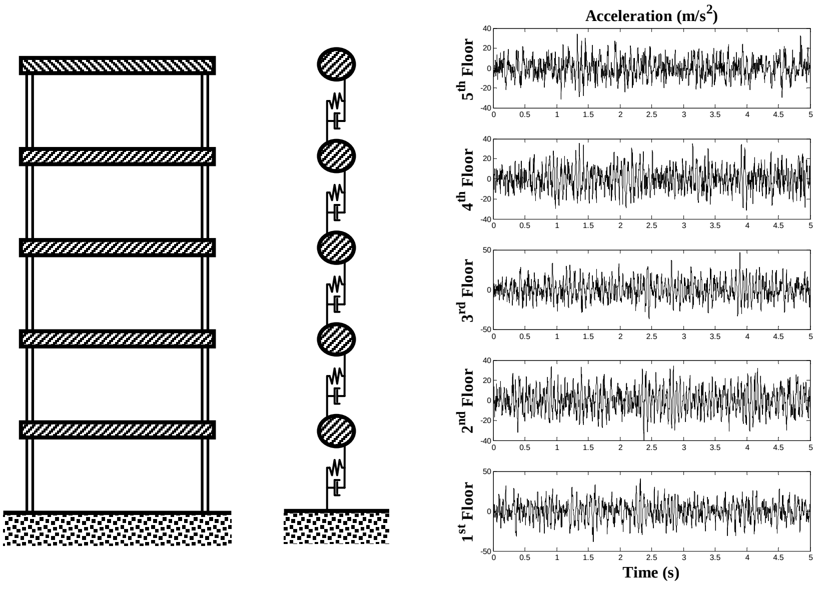

Figure 1: A five-story building modeled as a 5-MOF system

3. 1. Cepstral Metric as Damage Indicator

To illustrate the relationship between cepstral metric and damage occurred in structure as well its location and level, we consider five damage levels, 10%, 20%, 30%, 40% and 50%, of stiffness reduction on each story, hence total 25 damage scenarios. This structure can be modeled as a 5 degree-of-freedom (DOF) system, and the excitations

are filtered Gaussian white noise applied on every story. For each damage case, we simulated the dynamical responses of every DOF including acceleration, velocity, and displacement. In this paper, we only use the acceleration response, as shown in Figure 1, as time series for modeling despite the velocity and displacement can also be adopted.
The undamaged and unknown structures are referred to the reference and new models respectively, the distance between which is measured by the cepstral metric to determine if the unknown structure has been damaged, if so, to identify the location and level of damage. We have simulated $1+25=26$ datasets of acceleration time series, in which one is from the undamaged, and 25 are from the damaged of different scenarios. Consequently, for each damage scenario, we can obtain one corresponding cepstral metric, which is considered as the damage indicator, and hence totally obtain 25 cepstral metrics in different damage cases.
The cepstral metrics were calculated by the two approaches given in section 2. Figure 2 shows the cepstral metrics calculated by (10) and (16) using poles and subspace angles of AR models, respectively. From the figure, it is clear that the cepstral metric can detect the damage occurring in structure and tell which story the damage is near to. It is also clear that the magnitude of cepstral metric seems proportional to the damage level denoted by reduction of stiffness. Hence, the cepstral metric can act competently as an ideal damage indicator due to its change does strongly consist with damage both on location and level. Comparing these two cepstral metrics, the metric in terms of subspace angle performs a little better than that in terms of AR poles. However, it is not always be true since the calculation accuracy of cepstral metric depends on varieties of factors, such as identification method, the model order, and so on, although these two descriptions of cepstral metric are equivalent and they can be derived from each other.

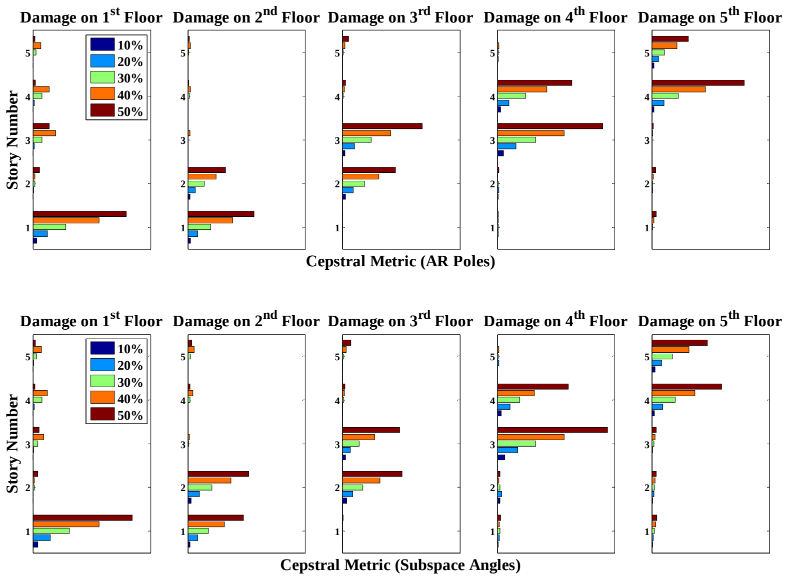

Figure 2: Cepstral metric from acceleration

3. 2. Interstory Acceleration

In this section, the acceleration of every story used above is replaced by the interstory acceleration, the difference of two adjacent stories, as the time series to build the model of dynamic system. The interstory acceleration is defined as following

$$ACC^{(i)}(t) = ACC^{(i)}(t) - ACC^{(i-1)}(t), \qquad (22)$$

where i is the story number, hence, we can obtain five new time series $ACC^{(1)}$, $ACC^{(2)} - ACC^{(1)}$, $ACC^{(3)} - ACC^{(2)}$, $ACC^{(4)} - ACC^{(3)}$ and $ACC^{(5)} - ACC^{(4)}$ for every damage case.

Figure 3 shows the cepstral metrics from the interstory accelerations by (10) and (16), respectively. Compared with the result of section 3.1, it is observed that the cepstral metric from the interstory acceleration is a better damage indicator since it is more clearly to tell which story is damaged according to its maximum value.

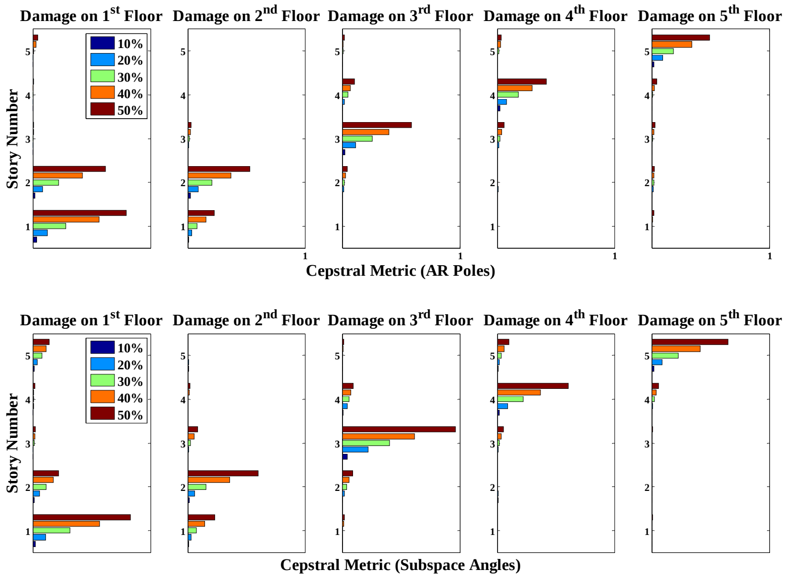

Figure 3: Cepstral metric from interstory acceleration

3. 3. Compare with Frequency Shift

The amount of literature related to damage detection using frequency shift is quite large due to the observation that changes in structural properties cause changes in vibration frequencies. It is known that for a multi-DOF vibration system, one single frequency shift associated with a certain mode does not provide the spatial information of structural damage. However, multiple frequency shifts can provide the spatial information of the damage location, and the magnitudes of frequency shifts proportionally imply the damage level.

The frequency shift is defined by $\Delta\omega_i / \omega_i$, where ω_i is the natural frequency of i-th mode for undamaged structure, and $\Delta\omega_i$ is the frequency change as a result of a certain damage scenario. Figure 4 shows the frequency shifts for the 25 different damage cases described in 3.1. It is clear that the frequency shift can't intuitively imply the damage location without the help of pattern recognition tools like support vector machine. Hence, the damage detection method using frequency shift as the damage indicator should be a two-step procedure. Alternatively, the cepstral metric, as the damage indicator, can directly succeed in doing so.

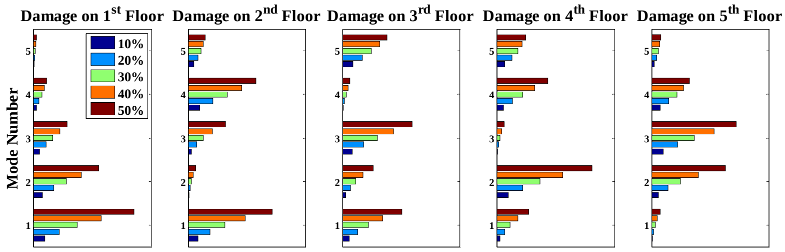

Figure 4: Frequency shift

4. CONCLUSIONS

In this paper, a new damage indicator, which is denoted by cepstral metric, is proposed for application of damage detection in civil engineering. This cepstral metric can be obtained by two approaches using either the poles and zeroes or the subspace angles of ARMA models, the equivalent relationship between which has been approved theoretically and numerically.

This methodology was tested on a numerical five-story shear building. Results show that the cepstral metric between the ARMA models from damage and undamaged structures can intuitively and clearly detect the damage location and damage level. It is worth noting that using interstory responses as time series for modeling can improve the performance of damage indicator especially for damage localization. Compared with other damage indicators for example the frequency shift, this methodology is a conveniently one-step procedure without the help of pattern recognition tool. These encouraging results indicate that the damage indicator expressed by cepstral metric is competent to be a sensitive damage indicator for structural health monitoring.

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